



## To analyse the curvature properties of a four-bar mechanism via Mathematical Modelling Techniques

Sanjana Saxena<sup>1</sup>, Pradeep Sharma<sup>2</sup>

<sup>1</sup>Mechanical Engineering Department, ABES Engineering College, Ghaziabad,

<sup>2</sup>Mechanical Engineering Department, ABES Engineering College, Ghaziabad,

**Abstract:** Analysis of the curvature properties of the link lengths of a four-bar mechanism via Mathematical Modeling Techniques using MATLAB Software. The proper identification of the lengths which has been taken analytically using Freudenstein's equation, which is an experimental work as well as virtual visualization of the work & the model of a 4-bar linkage, whose modeling has been done through MATLAB software. The objective of kinematic analysis is to optimize the best possible value of the combination of the link lengths & input/output angles of a four bar mechanism, when the input motion is given. It establishes the relationship between the motions of various components of the linkage.

**Keywords:** Curvature Properties, Four-bar, Freudenstein Equation, Mathematical Modelling, MATLAB.

### I. INTRODUCTION

Since ancient times mechanisms & machines have been used to reduce human effort and since the Industrial Revolution, they have entered and impacted almost all aspects of human society. Simply, mechanism is an assemblage of rigid links or bars connected by joints which shows the relative motions between the bars[5]. Input link of the mechanism is actuated and another output link can be made to perform a desired, intricate motion. The first well-known examples of the mechanism is the Watt's straight-line linkage. This mechanism was designed by James Watt. He invented the pull and push of the piston-rod in a double acting steam engine, who started the Industrial Revolution. Now-a-days mechanisms are present in a huge variety of gadgets, bottle cork openers, in bicycles, in garage door opening system, in steering and braking system of a car, in construction equipment for moving dirt and to move control surfaces of aircrafts, in spacecrafts to deploy solar panels and other appendages, in laparoscopic surgical tools, artificial prosthetic knees and other medical devices, to name a few. In the past 50 years, mechanisms have been combined with advanced electronics, sensors, control systems and computing technologies, and this marriage has resulted in devices such as robots, micro-electro mechanical systems (MEMS) and other so-called intelligent products.

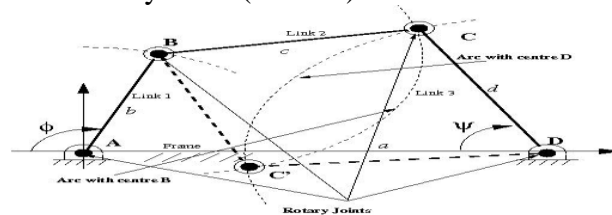


Figure 1: A four-link mechanism showing two possible configurations at a given  $\Phi$ [5]

**A. Analytical Approach of Freudenstein :** With reference to the graphical approach, Freudenstein developed an analytical approach for analysis and design of four bar link mechanisms. In his research, he presents an equation (equation 3) which relate the rotation angles  $\Phi$  and  $\psi$  in terms of the link lengths  $a$ ,  $b$ ,  $c$  and  $d$ . The scalar equation, which is now known as the Freudenstein equation, essentially is the condition for the loop closure assembly in a four bar link mechanism at a given  $\Phi$ . We follow the development of the Freudenstein equation using figure 2 which appears as figure 1 [5].

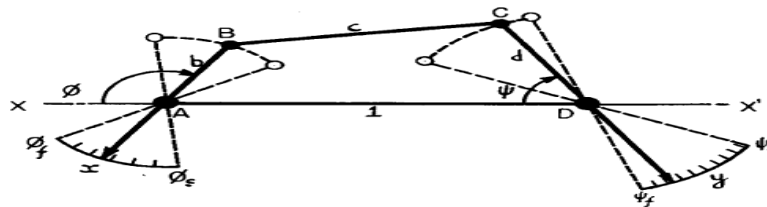


FIG. 1 FOUR-BAR LINKAGE

Fig2: A four-link mechanism for function generation [5]

In his research [4,5], the other lengths are denoted by  $b$ ,  $c$  and  $d$ , the frame is normalized to unity and the input and output angle are  $\Phi$  and  $\psi$ , respectively. The vector  $\mathbf{AB}$  locating point B with respect to A can be obtained in terms of  $b$  and angle  $\Phi$ , likewise the vector  $\mathbf{AC} = \mathbf{AD} + \mathbf{DC}$  can be obtained in terms of  $1$ ,  $d$  and angle  $\psi$ . Since the vector equation [5]

$$\mathbf{AB} + \mathbf{BC} = \mathbf{AD} + \mathbf{DC} \quad (1)$$

must be always satisfied to assemble the four-link mechanism, Freudenstein wrote the scalar equation

$$\mathbf{BC} \cdot \mathbf{BC} = (\mathbf{AB} + \mathbf{CD} + \mathbf{DA}) \cdot (\mathbf{AB} + \mathbf{CD} + \mathbf{DA}) \quad (2)$$

In the above equation, the vectors  $\mathbf{CD}$  and  $\mathbf{DA}$  are equal to the negative of  $\mathbf{DC}$  and  $\mathbf{AD}$ , respectively, and the symbol  $\cdot$  represents the vector dot product operation. Simplifying equation (2), Freudenstein obtained a simple scalar equation[5] .

$$R_1 \cos \Phi - R_2 \cos \psi + R_3 = \cos (\Phi - \psi) \quad (3)$$

where

$$R_1 = 1/d \quad (4)$$

$$R_2 = 1/b \quad (5)$$

$$R_3 = (1 + b^2 - c^2 + d^2)/(2bd) \quad (6)$$

Equation (3) is known as the *Freudenstein Equation*[5] and is readily applicable to kinematics analysis of four-bar mechanisms from known links lengths and the input angle  $\Phi$ , the output angle  $\psi$  can be found. Using the well known tangent half-angle trigonometric formulas for sine and cosine of angle  $\psi$ , it is possible to show that there are two possible  $\psi$ 's for a given angle  $\Phi$  – a fact consistent with the graphical results obtained earlier[5]. Equation (3) can also be directly used for three precision point synthesis for a function generating four-link mechanism. Given three values of input  $\Phi_i$ ,  $i=1, 2, 3$ , and the corresponding three values of output  $\psi_i$ ,  $i=1, 2, 3$ , one can substitute these angle pairs in equation (3) to obtain three *linear* equations in  $R_1$ ,  $R_2$  and  $R_3$ . Once  $R_1$ ,  $R_2$  and  $R_3$  are obtained from the solution of the linear equations, one can easily obtain the link lengths  $b$ ,  $d$ , and  $c$  from equations (4), (5) and (6) respectively[5]. It is interesting to compare the graphical approach and the analytical approach for three precision point design of a four-link mechanism. In the former the centre of a circle is to be determined from three points in a plane whereas in the latter three linear equations need to be solved – both are very straight forward! A second difference is in the choice of the design variables – in the graphical approach the starting rotation of output link  $\psi_s$  is determined from the construction (Note: in figure 2,  $\mathbf{C}_1$  is not the same as  $\mathbf{C}$ ) whereas in the analytical approach this is inherently assumed which in turn yields the three linear equations. Finally, in the analytical approach, the 2 solution of the linear equations may give negative values of  $R_1$  and  $R_2$ . Since the link lengths  $d$  and  $b$  cannot be negative,  $d$  and  $b$  must be thought of as vectors, and when  $R_1$ ,  $R_2$  are negative  $\pi$  must be added to the initial angles  $\psi_s$ ,  $\Phi_s$ , respectively[5]. For designing with larger number of precision points, Freudenstein introduced two new variables  $p_i$  and  $q_i$  denoting the rotation angles from unspecified and arbitrary starting positions  $\Phi_s$  and  $\psi_s$ . Setting  $\Phi = \Phi_s + p_i$  and  $\psi = \psi_s + q_i$ , equation (3) now can be written as[5] :

$$R_1 \cos(\Phi_s + p_i) - R_2 \cos(\psi_s + q_i) + R_3 = \cos[(\Phi_s + p_i) - (\psi_s + q_i)], i = 1, 2, 3, 4, 5 \quad (7)$$

The above equation (7) can be used for four and five precision point synthesis. In his thesis [2,5] and his paper [4,5], Freudenstein develops a detailed solution for function generation with four and five-precision point synthesis for a four-link mechanism. Finally, to extend the equation for six and seven precision-point synthesis, Freudenstein[5] introduced  $q_i = r_{\psi}(y_i - y_s)$ . Since the scale factors are unspecified, two new variables are added to equation (7) and Freudenstein[5] could achieve up to seven precision point synthesis for a function generating four-link mechanism. It may be noted that for more than three precision points *non-linear transcendental* equations (7) need to be solved – in fact solution of non-linear equations goes hand-in-hand with modern kinematics of mechanisms and machines!

In addition to finite precision point synthesis, Freudenstein[5] also derived detailed formulation to design four-link mechanisms when only one precision point together with a number of derivatives, such as velocity and acceleration, are prescribed. One can clearly see the power and elegance of the Freudenstein analytical approach when four or more precision points are to be used or when derivative information is required to be used – in the graphical approach, the geometry constructions become very complex whereas Freudenstein’s approach can be easily programmed in a computer. In the next section, we present a numerical example to illustrate the design of a four-link mechanism for function generation given three precision points.

## **II. LITERATURE SURVEY**

Ferdinand Freudenstein[5], worked upon the Approximate synthesis of four-bar linkages. In this, Formulas are presented for obtaining the characteristics of a four-bar linkage, designed to generate an arbitrary function approximately over a finite range. A number of methods of varying degrees of accuracy and complexity have been developed, enabling a designer to select the one best suited to his requirements. [6] Retheesh Kumar\*, Prof.Akash Mohanty\*\*worked upon the Design, Synthesis and Simulation Of Four Bar Mechanism For Eliminate The Plowing Depth Fluctuations In Tractors. In this, a four bar mechanism was designed for the specified performance output for a tractor.Looking on the soil hardness the plowing depth fluctuates,therefore,To induce the correct plowing depth an Electrical control system is introduced,which helps to regulate the plowing depth accurately.The main focus of this paper is to analyze Dimensional synthesis is performed supported Freudenstein’s Equation and Chebechev’s[7] formula in MATLAB and calculate the structural error of linkage mechanism by using Adams. Anees Ahmad†, Kenneth J Waldron[8] ,worked upon , Synthesis of adjustable planar 4-bar mechanisms. In this research, This paper outlines synthesis techniques for 4-bar linkages, having adjustable driven crank pivots, for different motion generation problems. All the possible maximal motion specifications for the two “phases”, consisting of different combination of the five positions, are enumerated. This leads to four classes of motion generation problems. Synthesis procedure for each class is presented. An example of the P1-P2-P3:P1-P4-P5 case is solved. The method of solution is analytical in nature, and, therefore well suited for use on a digital computer. Scott B. Nokleby Ron P. Podhorodesk [9] , worked upon , Optimization-Based Synthesis of Grashof Geared Five-Bar Mechanisms. In this, A Quasi-Newton optimization routine and Grashof criteria for geared five-bar mechanisms are used to develop a Grashof five-bar mechanism synthesis routine. Sequential transformations mapping Grashof mechanism parameters satisfying sub-type specific upper and lower constraints are used. Convergence criteria of: (i) objective function value change (ii) mechanism parameter change, and (iii) task satisfaction are used. These criteria combined with search restarts, ensure the synthesis of an acceptable mechanism.

## **III. METHODOLOGY**

The four-bar linkage is a planar closed loop linkage used extensively in mechanical machinery. This linkage has four coplanar bars that connect end to end with four revolute joints. In this example, we model a four-bar linkage using the Binary Link and Pivot Mount custom blocks(fig 3).



Figure3: A four bar mechanism Model [10]

**B. Modelling Approach :-** To model the fourbar linkage, we represent each physical component with a SimMechanics block(fig 4). The linkage in this example has five rigid bodies—three binary links and two pivot mounts—that connect in a closed loop through four revolute joints. Two of the binary links have one peg and one hole. The third binary link has two holes. The fourth link is implicit: the fixed distance between the two coplanar pivot mounts represents this link. We represent the binary links and pivot mounts using the custom library blocks that you created in previous examples. We represent the four revolute joints using four Revolute Joint blocks from the SimMechanics Joints library. The two pivot mounts connect rigidly to the world frame. For this reason, the implicit link acts as the ground link. Two Rigid Transform blocks provide the rigid connection between the two pivot mounts and the World frame. A translation offset in each Rigid Transform block displaces the two pivot mounts symmetrically along the world frame Y axis.



Figure4: Physical components of a 4 bar linkage mechanism-SimMechanics block [10]

To guide model assembly, we can specify the desired initial state for one or more joints in the model. To do this, we use the State Targets menu of the joint blocks. The state targets that we can specify are the joint position and velocity. These are angular quantities in revolute joints. We can specify state targets for all but one of the joints in a closed loop. Connect and name the blocks as shown in the figure. The base frame ports of the Rigid Transform blocks must connect to World Frame block ( fig 5) .

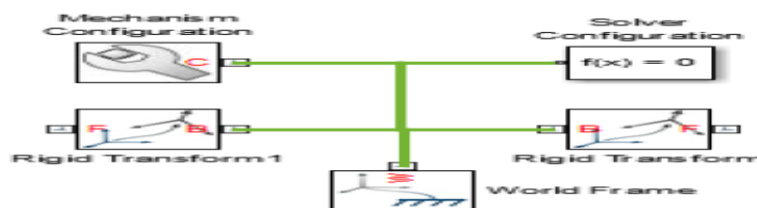


Figure 5: Base Frame [10]

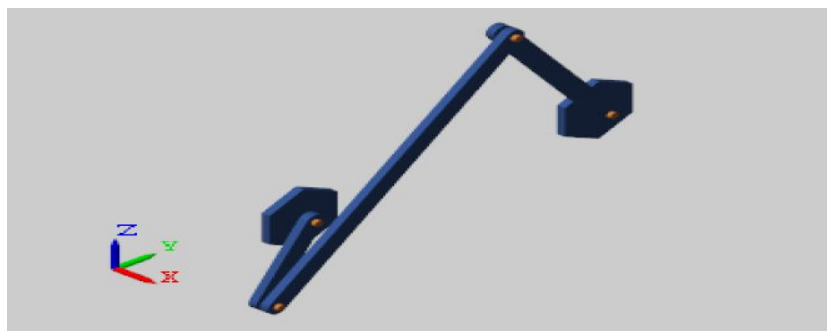


Figure 6: Solid Model of a 4 bar mechanism [10]

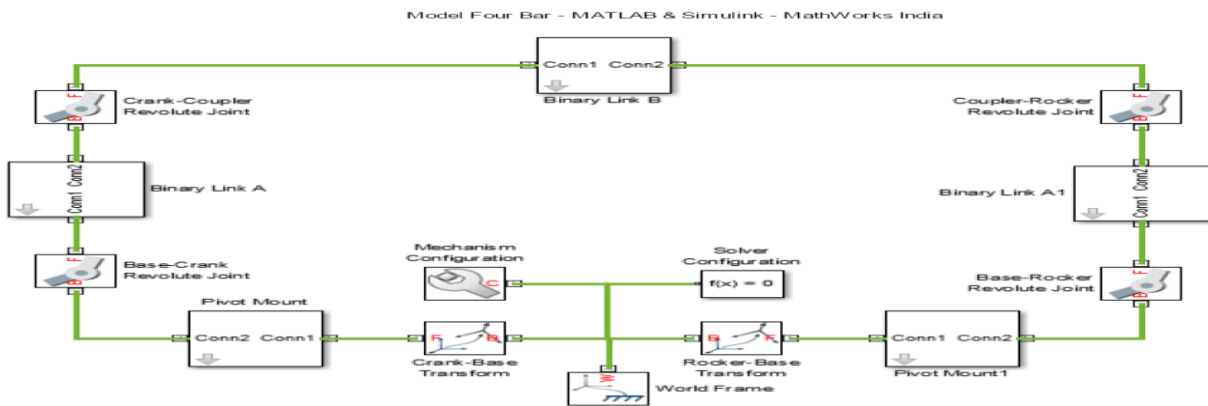


Figure 7: Model Four Bar MATLAB & Simulink MathWorks India [10]

We can guide assembly so that the four-bar linkage assembles in an open configuration instead(Fig 6). To do this, we must specify a position state target for at least one more joint. We do not have to specify this target precisely. If we have a general idea of what the target should be, we can enter an approximate value and select a low priority level for that target. Closed loop kinematic chains like the four-bar linkage are especially vulnerable to assembly issues. Even when the model assembles, SimMechanics may fail to meet one or more state targets. We can check the assembly status of the model and of the joints using the Model Report utility:

**C. Model Report :-** The figure shows the model report for the four bar linkage in the open configuration. A green circle indicates that SimMechanics satisfied the BaseCrank Revolute Joint state target precisely. A yellow circle indicates that SimMechanics satisfied the BaseRocker Revolute Joint state target approximately (fig 8).

Joint	Assembled	Primitive	Position				Priority	Status	Velocity				
			Actual	Specified	Unit				Actual	Specified	Units	Priority	Status
Base_Cran...	●	Rz	-30	-30	deg		High	●	+0		deg/s		
Base_Rock...	●	Rz	-5.33164	+0	deg		Low	▲	+0		deg/s		
Connecto...	●	Rz	+103.423		deg				+0		deg/s		
Crank_Co...	●	Rz	-78.7549		deg				+0		deg/s		

Figure 8: Model Report[10]

We have generated a code for the four-bar mechanism in MATLAB, & make a GUI , which shows the different relations between the different angles of a four bar mechanism , as shown in fig. The GUI formed , follows the Grashoffian criteria, and take the inputs as different lengths of the link . The curve generated in GUI shows the relation between the different angles(input & output ) with respect to time. The code generated is shown below:

```
function input_four_bars_main
clear all
global l1;global l2;global l3;global l4;global c;global err;global th;
err=0;
c=1;
l1=10;l2=5;l3=6;l4=6;
fig_1=figure('name','A.B.E.S ENGINEERING COLLEGE, GHAZIABAD - SANJANA
SAXENA','menubar','none','numbertitle',...
'off','color',[ 0.4898 0.4456 0.6463],'units',...
```

```

'normalized','position',[0,0,1,1]);
uicontrol('sty','text','un','n','pos',[.325 .88 .35 .1],'visible','on','string','Fourbar Mechanism',...
'fontsize',20,'BackgroundColor',[0.4898 0.4456 0.6463]);
K_p = uipanel(fig_1,...
'units','pixels',...
'Position',[350 30 100 200],...
'Title','Link lengths','FontSize',11,'BackgroundColor',[0.4898 0.4456 0.6463]);
rb = imread('fourbars_main.jpg');
image(rb); set(gca,'Position',[0.3 0.38 0.4 0.5]);
axis off;
uicontrol('sty','text','un','n','pos',[.22 .24 .05 .03],'visible','on','string','Link 1',...
'fontsize',15,'BackgroundColor',[0.4898 0.4456 0.6463]);
uicontrol('sty','text','un','n','pos',[.22 .19 .05 .03],'visible','on','string','Link 2',...
'fontsize',15,'BackgroundColor',[0.4898 0.4456 0.6463]);
uicontrol('sty','text','un','n','pos',[.22 .14 .05 .03],'visible','on','string','Link 3',...
'fontsize',15,'BackgroundColor',[0.4898 0.4456 0.6463]);
uicontrol('sty','text','un','n','pos',[.22 .09 .05 .03],'visible','on','string','Link 4',...
'fontsize',15,'BackgroundColor',[0.4898 0.4456 0.6463]);
uicontrol('sty','text','un','n','pos',[.57 .26 .17 .05],'visible','on','string','Mechanism Type',...
'fontsize',15,'BackgroundColor',[0.4898 0.4456 0.6463]);
uicontrol('sty','text','un','n','pos',[.42 .26 .17 .05],'visible','on','string','Driven Link Angle',...
'fontsize',15,'BackgroundColor',[0.4898 0.4456 0.6463]);
uicontrol('sty','text','un','n','pos',[.71 .01 .27 .05],'visible','on','string','Note: All lengths are in mm',...
'fontsize',11,'BackgroundColor',[0.4898 0.4456 0.6463]);
LD = 100; % Left, used to set the GUI.
HT = 25; % Height
BT = 140; % Bottom
K1=37;
l1_edit = uicontrol(K_p,'style','edit',...
'String',0,...
'callback',@l1_edit_button_press,...
'Position',[LD-75 BT 50 HT]); % L, B, W, H
l2_edit = uicontrol(K_p,'style','edit',...
'String',0,...
'callback',@l2_edit_button_press,...
'Position',[LD-75 BT-K1 50 HT]); % L, B, W, H
l3_edit = uicontrol(K_p,'style','edit',...
'String',0,...
'callback',@l3_edit_button_press,...
'Position',[LD-75 BT-2*K1 50 HT]); % L, B, W, H
6
l4_edit = uicontrol(K_p,'style','edit',...
'String',0,...
'callback',@l4_edit_button_press,...
'Position',[LD-75 BT-3*K1 50 HT]); % L, B, W, H
uicontrol('style','push','units','normalized','position',[.78 .2 .1
.05],'string','Simulation','fontsize',11,'callback',@simulation)
uicontrol('style','push','units','normalized','position',[.78 .1 .1
.05],'string','Help','fontsize',11,'callback',@help)
%mechanism1 = uicontrol('style','popupmenu','units','normalized','position',[.7 .5 .1
.1],'string',{'parallel';'anti_parallel'});%,...

```

```

% 'callback',@popp);
function l1_edit_button_press(h,dummy)
    l1 = str2double(get(h,'string'));
    if isnan(l1)
        errordlg('enter a numeric value')
    end
    end
    function l2_edit_button_press(h,dummy)
    l2 = str2double(get(h,'string'));
if isnan(l2)
    errordlg('enter a numeric value')
    end
    end
    function l3_edit_button_press(h,dummy)
    l3 = str2double(get(h,'string'));
    if isnan(l3)
        errordlg('enter a numeric value')
    end
    end
    end
function l4_edit_button_press(h,dummy)
l4 = str2double(get(h,'string'));
    if isnan(l4)
        errordlg('enter a numeric value')
    end
    end
    end
h1 = uibuttongroup('visible','off','Position',[0.6 0.11 .11 .13]);
    u0 = uicontrol('Style','Radio','String','Parallel','fontsize',11,...
    'pos',[10 43 100 40],'parent',h1,'HandleVisibility','off');
    u1 = uicontrol('Style','Radio','String','Anti Parallel','fontsize',11,...
    'pos',[10 3 100 40],'parent',h1,'HandleVisibility','off');
set(h1,'SelectionChangeFcn',@theta_choice);
    set(h1,'SelectedObject',u0);
    set(h1,'Visible','on');
    h2 = uibuttongroup('visible','off','Position',[0.45 0.11 .11 .13]);
    th0 = uicontrol('Style','Radio','String','Theta2','fontsize',11,...
    'pos',[10 43 100 40],'parent',h2,'HandleVisibility','off');
    th1 = uicontrol('Style','Radio','String','Theta3','fontsize',11,...
    'pos',[10 3 100 40],'parent',h2,'HandleVisibility','off');
    set(h2,'SelectionChangeFcn',@theta_choice);
    set(h2,'SelectedObject',th0); % No selection
    set(h2,'Visible','on');

    function theta_choice(varargin)
if get(u0,'value')==1
    c=1;
    end
    if get(u1,'value')==1
        c=-1;
    end
    if get(th0,'value')==1
        th=1;

```

```

        end
        if get(th1,'value')==1
            th=2;
    end
end
function check(varargin)
    V(1)=11; V(2)=12; V(3)=13; V(4)=14;
    shortest=min(V(1:4));
    longest=max(V(1:4));
    P=shortest + longest;
    Q=sum(V(1:4))- P;
    if P>=Q
        errordlg('Linkage is non Grashofian','Input Error')
        err=1;
    else
        err=0;
    end
    if 11==0 | 12==0 | 13==0 | 14==0
        err=1;
    end
end
function simulation(varargin)
    check
    if err==0
        go
    end
end
function help(varargin)
    f2=figure('name','Kinematic Analysis: Help','menubar','none','numbertitle',...
    'off','color',[ 0.4898 0.4456 0.6463],'units',...
    'normalized','position',[.1,.1,.8,.8]);
    uicontrol('sty','text','un','n','pos',[.4 .90 .1 .08],'visible','on','string','Help:',...
    'fontsize',22,'BackgroundColor',[0.4898 0.4456 0.6463]);
    uicontrol('sty','text','un','n','pos',[.045 .8 .9 .08],'visible','on','string','a) This code works only in case of
    grashoffian condition which states that sum of '
    'fontsize',15,'BackgroundColor',[0.4898 0.4456 0.6463]);
    uicontrol('sty','text','un','n','pos',[.045 .76 .865 .08],'visible','on','string','lengths of shortest and longest
    link is less than sum of other two links .',...
    'fontsize',15,'BackgroundColor',[0.4898 0.4456 0.6463]);
    uicontrol('sty','text','un','n','pos',[.045 .69 .88 .08],'visible','on','string','b) Double-crank,crank-rocker
    or double rocker are three possible mechanism .',

    'fontsize',15,'BackgroundColor',[0.4898 0.4456 0.6463]);
    uicontrol('sty','text','un','n','pos',[.045 .62 .66 .08],'visible','on','string','c) Simulations of respective
    mechanisms are shown .',
    'fontsize',15,'BackgroundColor',[0.4898 0.4456 0.6463]);
    uicontrol('sty','text','un','n','pos',[.045 .55 .785 .08],'visible','on','string','d) Plots of theta2 and theta3
    with respect to theta1 are also shown .',...
    'fontsize',15,'BackgroundColor',[0.4898 0.4456 0.6463]);
    uicontrol('sty','text','un','n','pos',[.045 .48 .795 .08],'visible','on','string','e) Enter the link lengths in
    check boxes according to grashoff criteria .',...

```



```

'fontsize',15,'BackgroundColor',[0.4898 0.4456 0.6463]);
uicontrol('style','push','units','normalized','position',[.5 .05 .1
.05],'string','Close','callback',@close_help)
uicontrol('sty','text','un','n','pos',[.045 .41 .645 .08],'visible','on','string','f) Choose either parallel or
anti option then click go .',...
'fontsize',15,'BackgroundColor',[0.4898 0.4456 0.6463]);
end
function close_help(varargin)
close 'Kinematic Analysis: Help'
end
function go(varargin)
V(1)=11; V(2)=12; V(3)=13; V(4)=14;
if 11==min(V(1:4)) | 14==min(V(1:4)) | 13==min(V(1:4))
if 13<11
temp=13;
l3=11;
l1=temp;
end
fourbars(l1,l2,l3,l4,c,th)
else
double_rocker(l1,l2,l3,l4,c,th)
end
end
clc
end

```

After Running this code, we entered the GUI & the following page is show in MATLAB, as shown in fig. The set of values has to be entered along with the driven theta angle & mechanism type & click the simulation (fig 9).

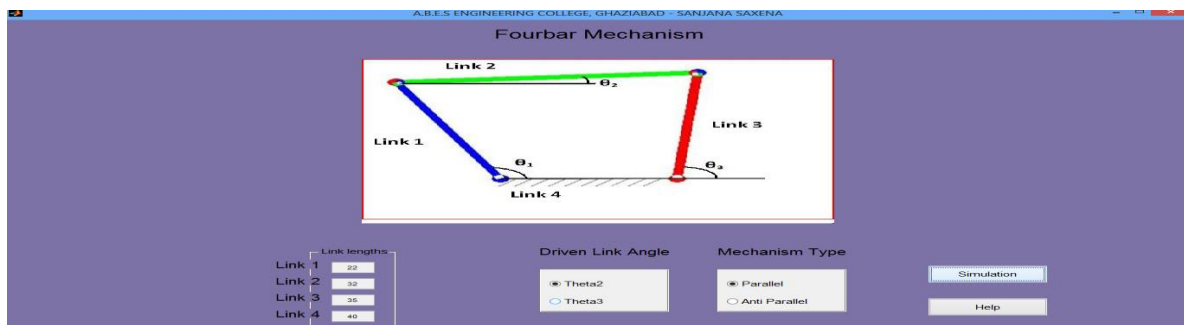


Figure 9 : GUI in MATLAB(Input link Lengths & corresponding theta)[10]

#### IV. RESULTS

After simulation, the driving four bar mechanism can be seen, with different curve plots shown between the different angles with time. The curve shown for the given input is as follows(fig 10)

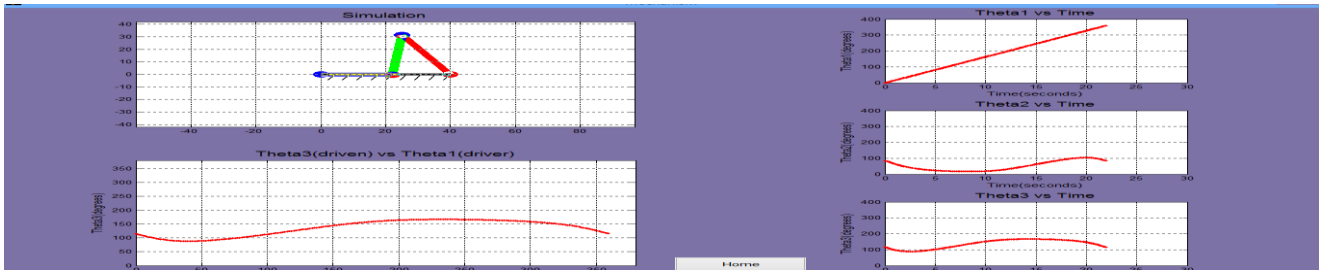


Figure 10: GUI in MATLAB[10]

- i) The entered values have the following assumptions [10]:
- ii) This code is work only in the Grashofian condition, which states that the sum of the shortest and the longest link is less than the other two links.
- iii) Double-crank, crank rocker or double rocker are the three possible mechanism.
- iv) Simulation of a different mechanism are shown.
- v) Plots of theta2 & thata3 with respect to theta1 are also shown.
- vi) Enter the link lengths in check boxes according to Grashof criteria.
- vii) Chose either parallel or anti option and press Go.

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