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A New Algorithm for Solving Fuzzy Transportation Problems with TrapezoidalFuzzy Numbers

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Abstract- In real world problems, optimization techniques are useful for solving problems like, project schedules, assignment problems and network flow analysis. The main aspect of this paper is to find the least transportation cost of some commodities through a capacitated network when the supply and demand of nodes and the capacity and cost of edges are represented as fuzzy numbers.
Here, we are proposing a new algorithm for solving fuzzy transportation problem, where fuzzy demand and supply all are in the form of trapezoidal fuzzy numbers. where fuzzy demand and supply all are in the form of trapezoidal fuzzy numbers. So the proposed approach is very easy to understand and to apply on real life transportation problems for the decision makers.
Key words: Trapezoidal fuzzy numbers, Fuzzy transportation problem, Ranking Technique.

I. INTRODUCTION

The Transportation problem is a special type of linear programming problem which deals with the distribution of single product (raw or finished) from various sources of supply to various destination of demand in such a way that the total transportation cost is minimized. There are effective algorithms for solving the transportation problems when all the decision parameters, i.e. the supply available at each source, the demand required at each destination as well as the unit transportation costs are given in a precise way. But in real life, there are many diverse situations due to uncertainty in one or more decision parameters and hence they may not be expressed in a precise way. This is due to measurement inaccuracy, lack of evidence, computational errors, high information cost, whether conditions etc. Hence we cannot apply the traditional classical methods to solve the transportation problems successfully. Therefore the use of Fuzzy transportation problems is more appropriate to model and solve the real world problems. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand are fuzzy quantities.

Bellman and Zadeh [3] proposed the concept of decision making in Fuzzy environment. After this pioneering work, several authors such as Shiang-Tai Liu and Chiang Kao[16], Chanas et al[5], Pandian et.al [14], Liu and Kao [11] etc proposed different methods for the solution of Fuzzy transportation problems. Chanas and Kuchta [4] proposed the concept of the optimal solution for the Transportation with Fuzzy coefficient expressed as Fuzzy numbers. Chanas, Kolodziejckzy, Machaj[5] presented a Fuzzy linear programming model for solving Transportation problem. Liu and Kao [11] described a method to solve a Fuzzy Transportation problem based on extension principle. Lin introduced a genetic algorithm to solve Transportation with Fuzzy objective functions.

NagoorGani and Abdul Razak [13] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. A.NagoorGani, Edward Samuel and Anuradha [7] used Arshamkhan's Algorithm to solve a Fuzzy Transportation problem. Pandian and Natarajan [14] proposed a Fuzzy zero point method for finding a Fuzzy optimal solution for Fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers.

In this paper, a new algorithm is proposed for solving a specialtype of fuzzy transportation problems. In the proposed algorithm transportation cost represented as trapezoidal fuzzy numbers. The method is to rank the fuzzy objective values of the objective function by some ranking method for numbers to find the best alternative. On the basis of this idea the ranking method with the help of α solution has been adopted a transform the fuzzy transportation problem. To illustrate the proposed algorithm a numerical example is solved and the obtained results are compared with the results of existing approaches. So the proposed approach is very easy to understand and to apply on real life transportation problems for the decision makers.

This paper is organized as follows: In section 2 deals with some basic terminology, section 3 provides the mathematical formulation of Fuzzy transportation problem, section 4 provides a proposed algorithm for solving fuzzy transportation problem, section 5 deals with the numerical example based on proposed algorithm.

II. TERMINOLOGY

In this section some basic definitions of fuzzy set theory are reviewed (Dubois and Prade, 1980), (Kauffman and Gupta, 1991).

2.1 Definition

The characteristic function $\mu_A(x)$ of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}(x)$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \rightarrow [0,1]$. The assigned value indicate the membership grade of the element in the set A. The function $\mu_{\tilde{A}}(x)$ is called the membership function and the set $\tilde{A} = \{(x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0,1]\}$. is called a fuzzy set.

2.2 Definition

A fuzzy set A, defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_A : R \rightarrow [0,1]$ has the following characteristics

(i) A is normal. It means that there exists an x ∈ R such that $\mu_A(x) = 1$

(ii) A is convex. It means that for every $x_1, x_2 \in R$,

$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}, \ \lambda \in [0,1]$

(iii) μ_A is upper semi-continuous.

(iv) supp (A) is bounded in R.

2.3 Definition A fuzzy number A is said to be non-negative fuzzy number if and only $\mu_{\tilde{A}}(x) = 0, \ \forall x < 0$

2.4 Definition

A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by, where a≤ b≤c≤d.

$$\mu_{\tilde{A}}(x)=\begin{cases}0, & x<a\\ \frac{x-a}{b-a}, & a<x\le b,\\ 1, & b<x<c,\\ \frac{d-x}{d-c}, & c\le x<d,\\ 0, & x>d\end{cases}$$

2.5 Definition

A trapezoidal fuzzy number $\tilde{A}=(a,b,c,d)$ is said to be non-negative (non positive) trapezoidal fuzzy number. i.e. $A\ge 0 (A\le 0)$ if and only if $a\ge 0 (c\le 0)$. A trapezoidal fuzzy number is said to be positive (negative) trapezoidal fuzzy number i.e. $A>0(A<0)$ if and only if $a>0(c<0)$.

2.6 Definition Two trapezoidal fuzzy number $\tilde{A}_1=(a,b,c,d)$ and $\tilde{A}_2=(e,f,g,h)$ are said to be equal. i.e. $\tilde{A}_1=\tilde{A}_2$ if and only if a=e, b=f, c=g, d=h.

2.7 Definition

Let $\tilde{A}_1=(a,b,c,d)$ and $\tilde{A}_2=(e,f,g,h)$ be two non-negative trapezoidal fuzzy number then

(i) $\tilde{A}_1\oplus\tilde{A}_2=(a,b,c,d)\oplus(e,f,g,h)=(a+e,b+f,c+g,d+h)$

(ii) $\tilde{A}_1-\tilde{A}_2=(a,b,c,d)-(e,f,g,h)=(a-h,b-g,c-f,d-e)$

(iii) $-\tilde{A}_1=-(a,b,c,d)=(-d,-c,-b,-a)$

(iv) $\tilde{A}_1\otimes\tilde{A}_2=(a,b,c,d)\otimes(e,f,g,h)=(ae,bf,cg,dh)$

(v) $\frac{1}{\tilde{A}}\cong\left(1/d,1/c,1/b,1/a\right)$

2.8 Ranking Technique

Several approaches for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for comparing the fuzzy numbers is by the use of ranking function based on their graded means. That is for every (a,b,c,d)∈F(R), the ranking function $\Re$:F(R) →R by graded mean is defined as $R(\tilde{A})=\left(\frac{a+b+c+d}{4}\right)$.

In this paper we use this method for ranking the objective values. The ranking index $R(\tilde{a})$ gives the representative value of fuzzy number $\tilde{a}$.

III. MATHEMATICAL FORMULATION OF A FUZZY TRANSPORTATION PROBLEM

Mathematically a transportation problem can be stated as follows:

Minimize

$$z=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij} \qquad \text{——(1)}$$

Subject to

$$\left.\begin{array}{lll} \sum_{j=1}^{n} x_{ij} = a_i & j = 1, 2, ..., n & \\ \sum_{i=1}^{m} x_{ij} = b_j & i = 1, 2, ..., m & \\ x_{ij} \geq 0 & i = 1, 2, ..., m, & j = 1, 2, ..., n \end{array}\right\} \quad ----(2)$$

Where c_{ij} is the cost of transportation of an unit from the ith source to the jth destination, and the quantity x_{ij} is to be some positive integer or zero, which is to be transported from the ith origin to jth destination. A obvious necessary and sufficient condition for the linear programming problem given in (1) to have a solution is that

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j \quad ——(3)$$

(i.e) assume that total available is equal to the total required. If it is not true, a fictitious source or destination can be added. It should be noted that the problem has feasible solution if and only if the condition (2) satisfied. Now, the problem is to determine x_{ij}, in such a way that the total transportation cost is minimum

Mathematically a fuzzy transportation problem can be stated as follows:

Minimize

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \quad ——(4)$$

Subject to

$$\left.\begin{array}{lll} \sum_{j=1}^{n} x_{ij} = \tilde{a}_i & j = 1, 2, ..., n & \\ \sum_{i=1}^{m} x_{ij} = b_j & i = 1, 2, ..., m & \\ x_{ij} \geq 0 & i = 1, 2, ..., m, & j = 1, 2, ..., n \end{array}\right\} \quad ----(5)$$

In which the transportation costs $\tilde{c}_{ij}$, supply a_i and demand $\tilde{b}_j$ quantities are fuzzy quantities. An obvious necessary and sufficient condition for the fuzzy linear programming problem give in (4-5) to have a solution is that

$$\sum_{i=1}^{n} a_i \square \sum_{j=1}^{m} \tilde{b}_j \quad ——(6)$$

This problem can also be represented as follows:

	1	………	n	Supply
1	$\tilde{c}_{11}$	………	$\tilde{c}_{1n}$	a_1
. . . .	. . . .	. . . .	. . . .	. . . .
m	$\tilde{c}_{m1}$	………	$\tilde{c}_{mn}$	a_m
Demand	$\tilde{b}_1$	………	$\tilde{b}_n$	

IV. PROPOSED ALGORITHM FOR SOLVING TRANSPORTATION PROBLEM

Following are the steps for solving Transportation Problem

Step 1: Select the first row (source) and verify which column (destination) minimum unit has cost. Write that source under column 1 and corresponding destination under column 2. Continue this process for each source. However if any source has more than one same minimum value in different destination then write all these destination under column 2.

Step 2: Select those rows under column-1 which have unique destination. For example, under column-1, sources are S1, S2, S3 have minimum unit cost which represents the destination D1, D1, D3 written under column 2. Here D3 is unique and hence allocate cell (S3, D3) a minimum of demand and supply. For an example if corresponding to that cell supply is 8, and demand is 6, then allocate a value 6 for that cell. However, if destinations are not unique then follow step 3. Next delete that row/column where supply/demand exhausted.

Step 3: If destination under column-2 is not unique then select those sources where destinations are identical. Next find the difference between minimum and next minimum unit cost for all those sources where destinations are identical.

*Step 4 :*Check the source which has maximum difference. Select that source and allocate a minimum of supply and demand to the corresponding destination. Delete that row/column where supply/demand exhausted.

Remark 1: For two or more than two sources, if the maximum difference happens to be same then in that case, find the difference between minimum and next to next minimum unit cost for those sources and select the source having maximum difference. Allocate a minimum of supply and demand to that cell. Next delete that row/column where supply/demand exhausted.

Step 5: Repeat steps 3 and 4 for remaining sources and destinations till (m+n-1) cells are allocated.

Step 6 : Total cost is calculated as sum of the product of cost and corresponding allocated value of supply/ demand. That is,

$$\text{Total Cost} = \sum\sum C_{ij} X_{ij}$$

V. NUMERICAL EXAMPLE

Consider the Fuzzy Transportation Problem

	FD_1	**FD_2**	**FD_3**	**FD_4**	**Fuzzy Capacity**
FO_1	[1,2,3,4]	[1,3,4,6]	[9,11,12,14]	[5,7,8,11]	[1,6,7,12]
FO_2	[0,1,2,4]	[-1,0,1,2]	[5,6,7,8]	[0,1,2,3]	[0,1,2,3]
FO_3	[3,5,6,8]	[5,8,9,12]	[12,15,16,19]	[7,9,10,12]	[5,10,12,17]
Fuzzy Demand	[5,7,8,10]	[1,5,6,10]	[1,3,4,6]	[1,2,3,4]	

Solution

In Conformation to model the fuzzy transportation problem can be formulated in the following mathematical programming form

Min Z = R(1,2,3,4)x_{11} + R(1,3,4,6)x_{12} + R(9,11,12,14)x_{13} + R(5,7,8,11)x_{14} +R(0,1,2,4)x_{21}+R(-1,0,1,2)x_{22}+R(5,6,78)x_{23}+R(4,5,6,7)x_{24}+R(3,5,6,8)x_{31}+R(5,8,9,12)x_{32} +R(12,15,16,19)x_{33} + R(7,9,10,12)x_{34}

$$R(1,2,3,4)=\frac{1+2+3+4}{4}=2.5$$

Similarly

R(1,2,3,4)=2.5; R(1,3,5,6)=3.75; R(9,11,12,14)=11.5; R(5,7,8,11)=7.75; R(0,1,2,4)=1.75; R(1,0,1,2)=0.5;R(5,6,7,8)=6.5; R(0,1,2,3)=1.5; R(3,5,6,8)=5.5;R(5,8,9,12)=8.5;R(12,15,16,19)=15.5; R (7,9,10,12)=9.5

Rank of all supply

R(1,6,7,12)= 6.5; R(0,1,2,3)= 1.5; R(5,10,12,17)=11

Rank of all demand

R(5,7,8,10)=7.5;R(1,5,6,10)=5.5;R(1,3,4,6) = 3.5; (1,2,3,4) = 2.5

Table after ranking

Table -1

	D1	**D2**	**D3**	**D4**	**Supply**
O1	2.5	3.5	11.5	7.75	6.5
O2	1.75	0.5	6.5	1.5	1.5
O3	5.5	8.5	15.5	9.5	11
Demand	7.5	5.5	3.5	2.5	

Step-1

The minimum cost value for the corresponding sources 01, 02, 03 is 2.5, 0.5, 5.5, which represents the destination D1, D2, D5 respectively which shown in Table -2

Table -2

Colum -1	**Colum – 2**
O1	D1
O2	D2
O3	D1

Step – 2

Hence the destinations D2 is unique for source O2 and allocate (O2, D2) in (1.5, 5.5)=1.5.

This is shown in Table – 3

Table - 3

	D1	**D2**	**D3**	**D4**	**Supply**
O1	2.5	3.5	11.5	7.75	6.5
O2	1.75	**1.5** 0.5	6.5	1.5	1.5
O3	5.5	8.5	15.5	9.5	11
Demand	7.5	**5.5** 4.00	3.5	2.5	19

Step – 3

Delete Row O2 as for this supply is exhausted and adjust Demand as (5.5 – 1.5) = 4.00.

Next the minimum cost value for the corresponding source O1, O3 are 2.5, 5.5 which represents D1, D1 respectively, which is shown in Table – 4

Table – 4

Colum -1	**Colum – 2**
O1	D1
O3	D1

Hence the destinations are not unique because sources O1,O3 have identical destination D1, D1. So we find the difference below minimum and next minimum unit cost for the sources O1, O3. The difference are 1, 3 respectively for the sources O1, O3.

Step-4

Hence the maximum difference is 3 which represents source O3. Now allocate the cell (O3, D1) min (11, 7.5) = 7.5 which shown in Table – 5

Table - 5

	D1	**D2**	**D3**	**D4**	**S**
O1	2.5	3.5	11.5	7.75	6.5
O3	**7.5** 5.5	8.5	15.5	9.5	11
D	7.5	4.00	3.5	2.5	19

Step – 5

Delete column D1, as demand is exhausted. The adjust supply as (C11-7.5) = 3.5.

Next the minimum unit cost for the corresponding sources O1, O3 are 3.5, 8.5 which represents the destination D2, D2 respectively which is shown in Table- 6

Table - 6

Colum -1	**Colum – 2**
O1	D2
O3	D2

Hence the destination are not unique because sources O1, O3 have identical destination of D2. So we find the difference between minimum and next minimum unit cost for the sources O1, O3. The differences are 4.25, respectively for the sources O1, O3.

Step – 6

Hence the maximum difference is 4. 25 which represents source O1. Now allocate the cell (O1,D2), min (6.5, 4) = 4 which shown in Table – 7

Table -7

	D2	**D3**	**D4**	**Supply**
O1	**4.00** / 3.5	11.5	7.75	6.5
O3	8.5	15.5	9.5	3.5
Demand	4.00	3.5	2.5	19

Step -7

Delete Column D2 as demand is exhausted next adjust supply as (6.5-4) = 2.5.

Next the minimum unit cost for the corresponding O1, O3 are 7.75 and 9.5 which represents the destination D4, D4 respectively which is shown in Table – 8

Table -8

Colum -1	**Colum – 2**
O1	D4
O3	D4

Hence the destination is not unique because sources O1, O3 have identical destination of D4. So we final the difference below minimum and next minimum unit cost for the sources.

The differences are 3.75, 6 respectively for the sources O1, O3.

Step – 8

Hence the maximum difference is 6 which represent sources O3. Now allocate the cell (O3, D4), min (3.5, 2.5) = 25 which is shown in Table – 9

	D3	**D4**	**Supply**
O1	11.5	7.75	2.5
03	15.5	**2.5** / 9.5	3.5
Demand	3.5	2.5	19

Step – 9

Delete column D4 as demand is exhausted. Next adjust supply as (3.5-2.5) = 1. Next the minimum unit cost for the corresponding sources O1, O3 are 11.5, 15.5 which represents the destination D3 , D3 respectively which is shown in Table – 10.

Table – 10

Colum -1	**Colum – 2**
O1	D3
O3	D3

Hence the sources O1, O3 have identical destination D3, So we must find minimum difference. However only one column remain and hence minimum difference can not be obtained. So allocate the remaining supply 2.5, 1 to cells (O1, D3), (103, D3) which shown in Table – 11.

Table – 11

	D3	**Supply**
O1	**2.5** 11.5	2.5
O3	**1** 15.5	1
Demand	3.5	

Step – 10

Hence (3+4-1)=6 cells are allocated and hence we got our feasible soln. Next we calculate total cost and its corresponding allocated value of supply demand which is shown in Table - 12

Table – 12

	D1	**D2**	**D3**	**D4**	**Supply**
O1	2.5	**4** 3.5	**2.5** 11.5	7.75	6.5
O2	1.75	**1.5** 0.5	6.5	1.5	1.5
O3	**7.5** 5.5	8.5	**1** 15.5	**2.5** 9.5	11
D	7.5	5.5	3.5	2.5	

Total Cost
(4x3.5)+(2.5x11.5)+(1.5x2.5)+(7.5x5.5)+(1x15.5)+(2.5x9.5)=14+28.75+3.75+41.25+15.5+23.7=127

This is a basic feasible solution. The solution obtained using NCM, LCM, VAM and MODI/Stepping stone methods respectively. Hence the basic feasible solution obtained from new method is optional soln.

Result:

Our Solution is same as that of optional solution obtained by using LCM, VAM, MODI/Stepping stone methods. Thus our method also gives optional soln.

VI. CONCLUSION

In this paper, the transportation costs are considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy transportation problem of trapezoidal fuzzy numbers has been transformed into crisp transportation problem using ranking indices. Numerical examples show that by this method we can have the optimal solution as well as the crisp and fuzzy optimal total cost. By using ranking method we have shown that the total cost obtained is optimal. Moreover, one can conclude that the solution of fuzzy problems can be obtained by our proposed method effectively. This technique can also be used in solving other types of problems like, project schedules, assignment problems and network flow problems.

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