PERFORMANCE ANALYSIS OF TWO-DEGREE-OF-FREEDOM CONTROLLER AND MODEL PREDICTIVE CONTROLLER FOR THREE TANK INTERACTING SYSTEM

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Abstract — Many processes in petrochemical industries, power plants and water treatment industries are nonlinear in nature. Also process industries utilizes three tank interacting system for processing of two different chemical composition streams into a required chemical mixture in the mixing reactor process, with monitoring and controlling of flow rate and level of chemical stream. The control of fluid level in tanks and flow between tanks is a basic problem in the process industries, which depends on the system dynamics and interacting characteristics. The Proportional-Integral (PI) controller is commonly used to control the level in process industries. In this paper, it is proposed to obtain the mathematical modeling of three tank interacting system and to design Two-Degree-of-Freedom (2DOF) controller with Coefficient Diagram Method (CDM) and Model Predictive Controller (MPC) for controlling the level in the tank-3 of three tank interacting system. The controller is simulated using MATLAB/ SIMULINK environment and the results confirm the effectiveness of the proposed system than the conventional methods.

Keywords — Three tank interacting system, SISO, Auto tuned PI controller, Two-Degree-of-Freedom controller (2DOF), Coefficient Diagram Method (CDM) and Model Predictive Control (MPC).

I. INTRODUCTION

The three tank interacting system is widely in many industrial process applications, such as in the wastewater treatment plant, petrochemical plant and oil & gas systems because it contributes for obtaining mixture of various chemicals during mixing process. The fluid level control of three tank interacting system is difficult because of its nonlinearity and an associated dead time. Due to the nonlinear nature of several chemical processes, interest in nonlinear feedback control has been steadily increasing. Conventional linear controllers can yield a satisfactory performance if the process is operated close to a nominal steady state or is fairly linear. Thus, there is a need to develop and implement feedback control schemes that takes the process non linearity in control calculations during controller design, which should act as a robust controller.

A simple and robust control strategy based on polynomial approach, namely Coefficient Diagram Method (CDM) is considered as a candidate to design the Two-Degree-of-Freedom controller. Model Predictive Controller (MPC) is an optimal control strategy based on numerical optimization. Future control inputs and future plant responses are predicted using a model and optimized at regular interval with respect to a performance index. The remaining contents of the paper are organized as follows. The section two presents the hardware description of the conical tank system. The section three explains the modelling of the system. The section four provides the PI controller design. The section five deals with the 2DOF controller with CDM design. The section six deals with the MPC...
design. The results are obtained and the comparison is done in the section seven. The section eight gives the conclusion and future enhancements.

II. HARDWARE DESCRIPTION

The main control objective is to maintain the level \( h_3 \) in tank-3 by manipulating the varying inflow rate \( q_1 \) of tank-1. The controller is designed for the three tank interacting system and implemented using MATLAB software. The three tank system consists of three cylindrical tanks of equal dimensions. The tanks are connected to each other through a manual valve. Three rotameter is fixed at the inlet of each tank to measure the flow rate. The level of the water in the three tank interacting system is quantified by means of the DPT. The quantified level of water in the form of current in the range of (4-20) mA is sent to the DAQ in which ADC converts the analog data to digital data and feed it to the PC. The PC acts as the controller and data logger.

The controller considers the process variable as feedback signal and finds the manipulated variable as the output based on the predefined set point. The DAC module of the DAQ converts this manipulated variable to analog form into (4-20) mA current signal. The I/P converter converts the current signal to pressure in the range of (3-15) psi, which regulates the flow of water into the tank-1 based on the output height of the tank-3. To enable various system configurations, vertical tanks are interconnected by valves, which allows us to work with one, two or three tanks, at a time. The block diagram of three tank interacting system is shown in Figure 1.

![Figure 1 Block diagram of three tank interacting system](image)

The configuration utilized in the three tank interacting system is Single Input Single Output (SISO). Also the physical setup of three tank interacting system can be used as single tank system, two tank interacting system for various study purpose in analyzing the process dynamics.

III. MATHEMATICAL MODELLING

Consider an interacting cylindrical three tank process system, with single input & single output (SISO configuration). The control objective is to maintain a level \( h_3 \) in tank-3 by manipulating the varying inflow rate \( q_1 \) of tank-1, shown in Figure 2.
Figure 2 Schematic diagram of three tank interacting system

Where,

- The volumetric flow rate into tank-1 is \( q_1 \) (cm\(^3\)/sec)
- The volumetric flow rate from tank-1 to tank-2 is \( q_{12} \) (cm\(^3\)/sec)
- The volumetric flow rate from tank-2 to tank-3 is \( q_{23} \) (cm\(^3\)/sec)
- The volumetric flow rate from tank-3 is \( q_4 \) (cm\(^3\)/sec)
- The height of the liquid level in tank-1 is \( h_1 \) (cm)
- The height of the liquid level in tank-2 is \( h_2 \) (cm)
- The height of the liquid level in tank-3 is \( h_3 \) (cm)
- Three tanks (1, 2 & 3) have the same cross sectional area \( A_1, A_2 & A_3 \) (cm\(^2\))
- The cross sectional area of interaction pipes are given by \( a_{12}, a_{23} & a_4 \) (cm\(^2\))
- The valve coefficients of interaction pipes are given by \( c_{12}, c_{23} & c_4 \)

According to Mass Balance Equation,
Accumulation = Input – Output
Applying Mass Balance Equation on Tank-1:
\[
A_1 \frac{dh_1(t)}{dt} = q_1(t) - a_{12}(t).c_{12}(t).\sqrt{2gh_1 - h_2}
\]  
(1)

Applying Mass Balance Equation on Tank-2:
\[
A_2 \frac{dh_2(t)}{dt} = a_{12}(t).c_{12}(t).\sqrt{2gh_1 - h_2} - a_{23}(t).c_{23}(t).\sqrt{2gh_2 - h_3}
\]  
(2)

Applying Mass Balance Equation on Tank-3:
\[
A_3 \frac{dh_3(t)}{dt} = a_{23}(t).c_{23}(t).\sqrt{2gh_2 - h_3} - a_4(t).c_4(t).\sqrt{2gh_3}
\]  
(3)

To understand the behavior of a process, a mathematical description of the dynamic behavior of the process has been developed. But unfortunately, the mathematical model of most of the physical processes is nonlinear in nature. Also most of the tools for analysis, simulation and design of the controllers, assumes the process dynamics is linear in nature. In order to bridge this gap, the linearization of the nonlinear model is often needed. This linearization is with respect to a particular operating point of the system. In this section, the linearization of the nonlinear mathematical behavior of a process are done using Taylor series method. Then, a control system is developed and designed based on the linear model.

\[
f(h, Q) = f(h_s - Q_s) + \frac{\partial f(h-h_s)}{\partial h} + \frac{\partial f(Q-Q_s)}{\partial Q}
\]  
(4)
The operating parameters of three tank interacting system are given in Table 1.

Table 1 Operating parameters of three tank interacting system

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>DESCRIPTION</th>
<th>VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>g</td>
<td>Gravitational force (cm²/sec)</td>
<td>981</td>
</tr>
<tr>
<td>a₁₂, a₂₃ &amp; a₄</td>
<td>Area of pipe (cm²)</td>
<td>3.8</td>
</tr>
<tr>
<td>h₁ₛ</td>
<td>Steady state water level of tank-1 (cm)</td>
<td>10</td>
</tr>
<tr>
<td>h₂ₛ</td>
<td>Steady state water level of tank-2 (cm)</td>
<td>7.5</td>
</tr>
<tr>
<td>h₃ₛ</td>
<td>Steady state water level of tank-3 (cm)</td>
<td>5</td>
</tr>
<tr>
<td>A₁, A₂ &amp; A₃</td>
<td>Area of Tank (cm²)</td>
<td>176.71</td>
</tr>
<tr>
<td>C₁₂</td>
<td>Valve coefficient of tank-1</td>
<td>1</td>
</tr>
<tr>
<td>C₂₃</td>
<td>Valve coefficient of tank-2</td>
<td>1</td>
</tr>
<tr>
<td>C₄</td>
<td>Valve coefficient of tank-3</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Based on the above operating parameters of three tank interacting system, we obtained the state equation and output equation (State Space Model) of the SISO configuration [8].

**STATE EQUATION**

\[
\begin{pmatrix}
    \frac{\dot{h}_1}{h_1} \\
    \frac{\dot{h}_2}{h_2} \\
    \frac{\dot{h}_3}{h_3}
\end{pmatrix} =
\begin{pmatrix}
    -0.3012 & 0.3012 & 0 \\
    0.3012 & -0.6024 & 0.3012 \\
    0 & 0.3012 & -0.3970
\end{pmatrix}
\begin{pmatrix}
    h_1 \\
    h_2 \\
    h_3
\end{pmatrix} +
\begin{pmatrix}
    0.00568 \\
    0 \\
    0
\end{pmatrix}(q_1)
\]  

**OUTPUT EQUATION**

\[y = h_3 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} + 0\]  

Based on the above state space model of the three tank interacting system, we obtain the transfer function of the system by using coding in the MATLAB software.

\[
\frac{H_3(s)}{Q_1(s)} = \frac{0.0005133}{s^3 + 1.301s^2 + 0.3587s + 0.008691}
\]  

The FOPDT model of interacting three tank process (SISO System) is approximated using two point methods from actual third order transfer function system. The two point method proposed by K.R. Sundaresan and P.R. Krishnaswamy [7], is used here. Their method uses the times that reach 35.3% and 85.3% of the open loop response.

\[
H_p(s) = \frac{0.0591 e^{-5s}}{36.79s+1}
\]  

Where,  
\[K = \text{process gain} = 0.0591; \tau = \text{time constant} = 36.79; t_d = \text{delay time} = 5\]

**IV. PI CONTROLLER DESIGN**

The PI controller consists of proportional and integral term. The proportional term changes the controller output proportional to the current error value. Large values of proportional term make the system unstable. The Integral term changes the controller output based on the past values of error.
So, the controller attempts to minimize the error by adjusting the controller output. The PI gain values are calculated by using the MATLAB auto tuning algorithm.

\[ u(t) = K_c \left( e(t) + \frac{1}{T_i} \int_0^t e(t) \, dt \right) \]  

\[ (9) \]

4.1 Auto Tuning of PI Controller

The PI Tuner allows to achieve a good balance between performance and robustness for either one or Two-Degree-of-Freedom PI controllers. PI Tuner is used to tune PI gains automatically in a Simulink model containing a PI Controller or PI Controller (2DOF) block. The PI Tuner considers as the plant all blocks in the loop between the PI Controller block output and input. The blocks in the plant can include nonlinearities. Because automatic tuning requires a linear model, the PI Tuner computes a linearized approximation of the plant in the model. This linearized model is an approximation to a nonlinear system, which is generally valid in a small region around a given operating point of the system. The obtained gain values of PI controller based on the auto tune method is proportional gain \( K_p = 2.2093 \) and integral gain \( K_i = 0.4504 \). The two tuning parameters of PI controller that interact in their influence must be balanced by the designer; the integral term increases the oscillatory or rolling behavior of the closed loop system. Because the two tuning parameters interact with each other, it can be challenging to arrive at “best” tuning values once the controller is placed in automatic.

V. 2DOF CONTROLLER WITH CDM DESIGN

The Coefficient Diagram Method (CDM) was developed and introduced by Prof. Shunji Manabe in 1998. This method is an algebraic approach which is applied to polynomial loop in the parameter space, where a special diagram called coefficient diagram which will give the necessary design information. The strength of CDM is its simplicity and robust controller for any plant under practical limitation.

The simplicity over controller structure made it powerful for systems with various uncertainties. In CDM, the design specifications parameters are equivalent time constant \( \tau \), stability indices \( \gamma_i \) [9]. These parameters have certain relationship with each other which is explained in the design part with the controller polynomials. Block diagram of CDM is shown in Figure 3.

Figure 3 Block diagram of CDM control system

5.1 Steps involved in CDM controller design

- Define the plant in the right polynomial form.
- Analyze the performance specifications and derive design specifications for CDM, i.e. \( \tau, \gamma_i \)
- Assume the controller polynomials in the simplest possible form. Express it in the left polynomial form.
• Derive the Diophantine equation, and convert it to Sylvester Form and solve for unknown variables. In above equation, \( C \) is the coefficient matrix, \( l_i \) and \( k_i \) are the controller design parameters, and \( a_i \) are the coefficients of the target characteristic polynomial.

• Obtain the coefficient diagram of the closed-loop system and make some adjustments to satisfy the performance specifications if necessary [6].

5.2 Computation of 2DOF controller with CDM

**Step1:** FOPDT model of three tank interacting system is represented as

\[
G_P(s) = \frac{0.0519 e^{-5s}}{36.79 s+1} \quad (10)
\]

**Step 2:** Equivalent transfer function of the above said FOPTD model using first order Pade’s approximation technique is

\[
G_P(s) = \frac{N(s)}{D(s)} = \frac{-0.2955s+0.1182}{183.95s^2+78.58 s+2} \quad (11)
\]

**Step 3:** Two-Degree-of-Freedom CDM controller polynomials (\( B(s) \) and \( A(s) \)) are chosen as

\[
B(s) = K_2 s^2 + K_1 s + K_0; \quad A(s) = l_2 s^2 + l_1 s. \quad (12)
\]

**Step 4:** Selected stability indices values (\( \gamma_1, \gamma_2, \gamma_3 \)) are \( \gamma_1 = 2.5; \gamma_2 = 2.5; \gamma_3 = 2.0 \)

**Step 5:** Go to STEP 3 and obtain the controller design

\[
\frac{B(s)}{A(s)} = \frac{4687.67s^2+346.07 s+8.460}{65.780s^2+0.401s} \quad (13)
\]

The proposed 2DOF controller with CDM technique can be implemented by substituting the value of controller parameters in the control structure [8].

VI. MPC CONTROLLER DESIGN

Model predictive controller (MPC) started to emerge industrially in the 1980s as Identification and Command (IDCOM) by Richaletet.al. and Dynamic Matrix Control (DMC) by Cutler and Ramaker. The initial IDCOM and MPC algorithms represented the first generation of MPC technology. Generally, MPC is a family of controllers in which there is a direct use of an explicit identifiable model.

6.1 Model Predictive Control Strategy

• The overall objectives of an MPC controller have been summarized as follows
• Prevent violations of input and output constraints.
• Drive some output variables to their optimal set points, while maintaining other outputs with in specified ranges
• Prevent excessive movement of the input variables. Also it controls as many process variables as possible when a sensor or actuator is not available. The block diagram given in Figure 4 represents the MPC strategy.
6.2 Model Predictive Controller Design

In designing the MPC Controller, the MPC Toolbox from SIMULINK is used to access the MPC block. This MPC block mask has SIMULINK signals that are connected to the MPC Controller block's in ports. The measured output (mo) and reference (ref) in ports are required. The optional in ports can be chosen by selecting check boxes at the bottom of the mask. For this paper, simple linear model of MPC with an assumption of there is no disturbance (md) or no constraint is used. In designing the MPC controller, the parameters that required being set are sampling time (ts), control horizon (m) and prediction horizon (p). These parameters are important to have better performance of MPC when used to control the level of liquid in the tank [4]. The parameters for MPC Controller for this experiment are: Sampling time = 1 sec; Prediction horizon = 80 sec; Control horizon = 60 sec. The best parameter found above is using trial and error procedure since there is no standard method to find the optimum parameter.

VII. RESULTS AND DISCUSSION

7.1 Closed Loop Response of Auto Tuned PI Controller

The PI Tuner allows to achieve a good balance between performance and robustness for either one or Two-Degree-of-Freedom PI controllers. PI Tuner is used to tune PI gains automatically in a Simulink model containing a PI Controller. The PI controller attempts to minimize the error by adjusting the controller output. Closed loop response Auto Tuned PI controller is shown in Figure 5.

![Figure 5 Closed response of Auto Tuned PI controller](image-url)
the output, this may lead to unsatisfactory performance in real time application. To avoid such issues, we need to depend on the advanced control techniques for better performance in real time implementation.

7.2 Closed Loop Response of 2DOF Controller with CDM
The closed loop response is obtained using the Two-Degree-of-Freedom controller (2DOF) with CDM is shown in Figure 6.

![Figure 6 Closed response of 2DOF CDM controller](image)

From the Figure 6, it is evident that the response of the 2DOF controller with CDM takes quicker time in achieving the steady state for the given set point. Also the response has no overshoot in the output; this may lead to satisfactory performance in real time application. Based on this design the robustness of the system is increased and provides stable environment under operation.

7.3 Closed Loop Response of MPC
The closed loop response is obtained using the Model Predictive Control (MPC) is shown in Figure 7.

![Figure 7 Closed response of MPC](image)

From the Figure 7, it is evident that the response of the MPC controller takes quicker time in achieving the steady state for the given set point. It is found that, MPC provides better performance
in reduced settling time for properly tuned parameters but overshoot is present. Based on IAE, the MPC has minimum value compared to other controllers. Since peak overshoot of 4% in response is present, this could be avoided by increasing the weights with proper tuning.

7.4 Comparative Closed Loop Response of Proposed Controller

The comparative closed loop response of proposed controller with conventional controller is shown in the Figure 8.

![Comparative closed loop response of proposed controller](image)

Based on the above response of comparative closed loop response, the performance metrics is calculated and given in the below Table 2.

Table 2 Comparative performance metrics analysis of proposed controller

<table>
<thead>
<tr>
<th>PERFORMANCE METRICS</th>
<th>PI</th>
<th>2DOF-CDM</th>
<th>MPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time (sec)</td>
<td>90</td>
<td>100</td>
<td>31.3</td>
</tr>
<tr>
<td>Settling Time (sec)</td>
<td>470</td>
<td>110</td>
<td>100</td>
</tr>
<tr>
<td>Peak Time (sec)</td>
<td>135</td>
<td>120</td>
<td>40</td>
</tr>
<tr>
<td>Peak Overshoot (%)</td>
<td>16</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Integral Absolute Error (IAE)</td>
<td>1258</td>
<td>852</td>
<td>343</td>
</tr>
</tbody>
</table>

From the Table 2, the comparative analysis of controller performance based on the rise time, settling time, peak time, peak overshoot and integral absolute error of time domain response of the proposed system. By comparing the performance of 2DOF with CDM with conventional controller, the 2DOF with CDM has better response because it does not contain overshoot in output response and also the IAE value is 852 which is low when compared to the value of Conventional Controller. Therefore 2DOF with CDM has better and robust response in output, which is verified in the simulation result. Based on the comparison between 2DOF with CDM and MPC, the MPC has earlier rise time of 31.3 seconds, faster settling time of 100 seconds and 343 minimum IAE value when compared to the values of 2DOF with CDM. Since peak overshoot of 4% in response is present, this could be avoided by increasing the weights with proper tuning. To conclude that, the overall performance of MPC is faster response in the closed loop response based on the given set point. Therefore MPC performance is better than the other controllers.
VIII CONCLUSION

The three tank interacting system is highly non-linear process because of interaction between
the tanks. The controlling of nonlinear process is a challenging task. In this paper, the linearized
model of three tank interacting system is obtained. The Conventional PI controller, Two-Degree-of-
Freedom controller using CDM technique and Model Predictive Controller (MPC) are designed and
simulated using MATLAB. Based on the above proposed controller performance and from the
simulation results it has been found that Model Predictive Controller (MPC) provides the better
performance than its counterpart. In future, this study can be extended to implement the MPC with
input and output constraints in controller design of three tank interacting system. To enhance the
controller performance, optimization or adaptive techniques can be used for parameter tuning to
improve the performance of the MPC. The performance of the MPC can be further improved by
using Neural Network based model instead of step response model.

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BIOGRAPHIES

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