Some Topological Separation Axioms Using $Q^*g$ - Open Sets

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Abstract — In this paper, we introduce $Q^*g$ - $H_i$ - spaces $(i = 0, 1, 2)$ and $Q^*g$ - $U_i$ - Spaces $(i = 0, 1)$ in topological spaces and study its properties.

Keywords — $Q^*g$ - $H_i$ - Spaces $(i = 0, 1, 2)$ and $Q^*g$ - $U_i$ - Spaces $(i = 0, 1)$.

I. INTRODUCTION

Topology has a vital role in pure mathematics and has many subfields. The topology structured the foundation for geometry and algebra. There is no universal agreement among mathematicians as what a first course in topology should include. There are many topics that are appropriate to such a course and not all are equally relevant to the varied purposes.

Separation axioms are properties by which the topology on a space $X$ separates points from points, points from closed sets and closed sets from each other. The various separation axioms give rise to a sequence of successively stronger requirements, which are put upon the topology of a space to separate varying types of subsets.

In 1963, Levine introduced the concept of semi-open sets. Since then, a considerable number of papers discussing separation axioms, essentially by replacing open sets by semi-open sets, have appeared in the literature. For instance, Maheshwari and Prasad introduced semi-$T_0$, semi-$T_1$, semi-$T_2$, $s$ - normality and $s$ - regularity as a generalization of $T_0$, $T_1$, $T_2$, regularity and normality axioms respectively, and investigated their properties. The notion of semi-open sets was used by Maheshwari and Prasad to introduce pairwise semi-$T_0$, pairwise semi-$T_1$, pairwise semi-$T_2$, pairwise $s$ - regular and pairwise $s$-normal spaces. Moreover, $s$ - normal (resp. semi normal ) spaces were introduced and studied by Maheshwari and Prasad [11] (resp. Dorsett [9]). The notion of $Q^*$ - open sets in a topological space was introduced by Murugalingam and Lalitha [16, 17]. In the year 2015, P.Padma [20] introduced $Q^*g$ - closed sets in topological spaces.

Throughout this paper $X$ and $Y$ always represent nonempty bitopological spaces $(X, \tau)$ and $(Y, \sigma)$. In this paper, we introduce $Q^*g$ - $H_i$ - spaces $(i = 0, 1, 2)$ and $Q^*g$ - $U_i$ - Spaces $(i = 0, 1)$ in topological spaces and study its properties.

II. $Q^*G$ - $H_i$ - SPACES $(i = 0, 1, 2)$ AND $Q^*G$ - $U_i$ - SPACES $(i = 0, 1)$

Replacing open sets by $Q^g$- open sets and ‘cl’ by ‘$Q^g$cl’ in $H_i$ - spaces, $(i = 0, 1, 2)$ and $U_i$ - Spaces $(i = 0, 1)$ of Csaszar [6], we introduce $Q^*g$ - $H_i$ - spaces $(i = 0, 1, 2)$ and $Q^*g$ - $U_i$ - Spaces $(i = 0, 1)$.

Definition 2.1 [16] - A subset $A$ of a topological space $(X, \tau)$ is called a $Q^*$ - closed if $\text{int}(A) = \phi$ and $A$ is closed.

Example 2.1: Let $A = [0, 1)$. In the $\mathcal{U}$ topology $\text{int}(A) = (0, 1)$. However in the $\mathcal{U}$ topology $\text{int}(A) = [0, 1)$ w.r.to the $\mathcal{C}$ topology, $\text{int}(A) = \phi$. Hence $A$ is $Q^*$ - closed.

Definition 2.2 : A space $X$ is said to be $Q^g - R_0$ if for every pair of points $x$ and $y$ such that $x \not\in Q^*g cl\{y\}$ implies that $y \not\in Q^*g cl\{x\}$.
Definition 2.3: A space $X$ is said to be $Q^*_g - R_1$ if for every pair of distinct points $x, y$ of $X$ with $Q^*_g \text{cl} \{x\} \neq Q^*_g \text{cl} \{y\}$ there exists a $Q^*_g$-open set $U$ and a $Q^*_g$-open set $V$ such that $x \in U, y \in V, U \cap V = \phi$.

Definition 2.4: A space $X$ is said to be $Q^*_g - H_0$ if for every pair of points $x$ and $y$ such that $x \notin Q^*_g \text{cl} \{y\}$ there exists a $Q^*_g$-open set $U$ and a $Q^*_g$-open set $V$ such that $x \in U, y \in V, U \cap V = \phi$.

Definition 2.5: A space $X$ is said to be $Q^*_g - H_1$ if for every pair of points $x$ and $y$ such that $Q^*_g \text{cl} \{x\} \cap Q^*_g \text{cl} \{y\} = \phi$, there exists a $Q^*_g$-open set $U$ and a $Q^*_g$-open set $V$ such that $x \in U, y \in V, U \cap V = \phi$.

Definition 2.6: A space $X$ is said to be $Q^*_g - H_2$ if for every $Q^*_g$-closed set $A$ and a point $x$ such that $Q^*_g \text{cl} \{x\} \cap A = \phi$, there exists a $Q^*_g$-open set $U$ and a $Q^*_g$-open set $V$ such that $x \in U, A \subseteq V, U \cap V = \phi$.

Definition 2.7: A space $X$ is said to be $Q^*_g - U_0$ if for every pair of points $x$ and $y$ such that $x \notin Q^*_g \text{cl} \{y\}$, there exists a $Q^*_g$-open set $U$ and a $Q^*_g$-open set $V$ such that $x \in U, y \in V, Q^*_g \text{cl} \{U\} \cap Q^*_g \text{cl} \{V\} = \phi$.

Definition 2.8: A space $X$ is said to be $Q^*_g - U_1$ if for every pair of points $x$ and $y$ such that $Q^*_g \text{cl} \{x\} \cap Q^*_g \text{cl} \{y\} = \phi$, there exists a $Q^*_g$-open set $U$ and a $Q^*_g$-open set $V$ such that $x \in V, y \in U, Q^*_g \text{cl} \{U\} \cap Q^*_g \text{cl} \{V\} = \phi$.

Theorem 2.7: Every $Q^*_g$-normal space is $Q^*_g - H_2$.

Proof: Let $X$ is $Q^*_g$-normal space. Let $x \in X$ and let $A$ be a $Q^*_g$-closed set such that $Q^*_g \text{cl} \{x\} \cap A = \phi$. By $Q^*_g$-normality of $X$, there exists a $Q^*_g$-open set $U$ and a $Q^*_g$-open set $V$ such that $Q^*_g \text{cl} \{x\} \subseteq V, A \subseteq U, U \cap V = \phi$. Hence $x \in V, A \subseteq U, U \cap V = \phi$. Hence, $X$ is $Q^*_g - H_2$.

Theorem 2.8: Every $Q^*_g - H_2$ space is $Q^*_g - H_1$.

Proof: Let $X$ is $Q^*_g - H_2$ space. Let $x$ and $y$ be two distinct points of $X$ such that $Q^*_g \text{cl} \{x\} \cap Q^*_g \text{cl} \{y\} = \phi$. Since $X$ is $Q^*_g - H_2$, therefore there exists a $Q^*_g$-open set $U$ and a $Q^*_g$-open set $V$ such that $x \in V, Q^*_g \text{cl} \{y\} \subseteq U, U \cap V = \phi$. Thus $x \in V, y \in U, U \cap V = \phi$. Hence, $X$ is $Q^*_g - H_1$.

Theorem 2.9: Every $Q^*_g - H_0$ space is $Q^*_g - H_1$.

Proof: Let $X$ is $Q^*_g - H_0$ space. Let $x$ and $y$ be two distinct points of $X$ such that $Q^*_g \text{cl} \{x\} \cap Q^*_g \text{cl} \{y\} = \phi$. Hence $x \notin Q^*_g \text{cl} \{y\}$. Since $X$ is $Q^*_g - H_2$ therefore there exists a $Q^*_g$-open set $U$ and a $Q^*_g$-open set $V$ such that $x \in V, y \in U, U \cap V = \phi$. Hence, $X$ is $Q^*_g - H_1$.

Theorem 2.10: Every $Q^*_g - R_1$ space is $Q^*_g - H_0$.

Proof: Let $X$ is $Q^*_g - R_1$ space. Let $x \notin Q^*_g \text{cl} \{y\}$. Then $Q^*_g \text{cl} \{x\} \neq Q^*_g \text{cl} \{y\}$. Thus there exists a $Q^*_g$-open set $U$ and a $Q^*_g$-open set $V$ such that $x \in U, y \in V, U \cap V = \phi$. Hence, $X$ is $Q^*_g - H_0$.

Theorem 2.11: Every $Q^*_g - R_1$ space is $Q^*_g - H_1$.

Proof: Follows in view of Theorem 2.9 and 2.10.

Definition 2.9: A space $X$ is said to be strongly $Q^*_g$-regular if for each $Q^*_g$-closed subset $A$ of $X$ and $x \notin A$, there exist disjoint $Q^*_g$-open sets $U$ and $V$ such that $x \in U$ and $A \subseteq V$.

Theorem 2.12: Every $Q^*_g - H_0$ space is $Q^*_g - R_0$. 
Proof: Let X is $Q^g - H_0$ space. Let $x \in G \in Q^g - O (\tau)$ and let $y \in X - G$. Then $x \notin Q^g - cl \{y\}$. Since X is $Q^g - H_0$, there exists a $Q^g - open$ set U and a $Q^g - open$ set V such that $x \in U$, $y \in V$, $U \cap V = \phi$. Thus $\{x\} \cap V = \phi$ so that $y \notin Q^g - cl \{x\}$. Hence $X - G \subseteq X - Q^g - cl \{x\}$ or $Q^g - cl \{x\} \subseteq G$. Thus $X$ is $Q^g - R_0$.

Theorem 2.13: Every strongly $Q^g$ - regular space is $Q^g - H_2$.
Proof: Let X is strongly $Q^g$ - regular space. Let $x \in X$ and let A be a $Q^g - closed$ subset of X such that $Q^g - cl \{x\} \cap A = \phi$. Then $x \notin A$. By strongly $Q^g$ - regularity of X , there exists a $Q^g - open$ set U and a $Q^g - open$ set V such that $x \in U$, $y \in V$, $U \cap V = \phi$. Hence, X is $Q^g - H_2$.

Theorem 2.14: A space is $Q^g - T_2$ if and only if it is $Q^g - T_0$ and $Q^g - H_0$.
Proof: Let X is $Q^g - T_2$ space. Clearly, X is $Q^g - T_0$ and $Q^g - H_0$. Conversely, let X is $Q^g - T_0$ and $Q^g - H_0$. Let x, y be two distinct points of X. Then X is $Q^g - open$ set U or a $Q^g - open$ set V such that $x \notin U$, $y \notin V$. Thus $x \notin Q^g - cl \{y\}$ or $y \notin Q^g - cl \{x\}$. Since the space is $Q^g - H_0$, there exists a $Q^g - open$ set P and a $Q^g - open$ set Q such that $x \in P$, $y \in Q$. Hence, X is $Q^g - T_2$.

Theorem 2.15: A space is $Q^g - T_2$ if and only if it is $Q^g - T_1$ and $Q^g - H_1$.
Proof: Let X is $Q^g - T_2$ space. Clearly, X is $Q^g - T_1$ and $Q^g - H_1$. Conversely, let X is $Q^g - T_1$ and $Q^g - H_1$. Let x, y be two distinct points of X. Then X is $Q^g - open$ set U or a $Q^g - open$ set V such that $x \in U$, $y \in V$. Thus $x \notin Q^g - cl \{y\}$ or $y \notin Q^g - cl \{x\}$. Since the space is $Q^g - H_1$, there exists a $Q^g - open$ set U and a $Q^g - open$ set V such that $x \in U$, $y \in V$, $U \cap V = \phi$. Hence, X is $Q^g - T_2$.

Theorem 2.16: Every strongly $Q^g$ - regular space is $Q^g - U_0$.
Proof: Let X is strongly $Q^g$ - regular space. Let x, y \in X such that $x \notin Q^g - cl \{y\}$. Since the space is strongly $Q^g$ - regular, there exists a $Q^g - open$ set U and a $Q^g - open$ set V such that $x \in U$, $Q^g - cl \{y\} \subseteq V$, $Q^g - cl (U) \cap Q^g - cl (V) = \phi$. Hence $x \in U$, $y \in V$, $Q^g - cl (U) \cap Q^g - cl (V) = \phi$ and thus the space is $Q^g - U_0$.

Theorem 2.17: Every $Q^g - U_0$ space is $Q^g - H_0$.
Proof: Let X is $Q^g - U_0$ space. Let x, y \in X such that $x \notin Q^g - cl \{y\}$. Since X is $Q^g - U_0$, there exists a $Q^g - open$ set U and a $Q^g - open$ set V such that $x \in U$, $y \in V$, $Q^g - cl U \cap Q^g - cl V = \phi$. Hence $x \in U$, $y \in V$, $U \cap V = \phi$ and thus X is $Q^g - H_0$.

Theorem 2.18: Every $Q^g - U_1$ space is $Q^g - H_1$.
Proof: Let X is $Q^g - U_1$ space. Let x, y \in X such that $Q^g - cl \{x\} \cap Q^g - cl \{y\} = \phi$. Since X is $Q^g - U_1$, there exists a $Q^g - open$ set U and a $Q^g - open$ set V such that $x \in U$, $y \in V$, $Q^g - cl U \cap Q^g - cl V = \phi$. Hence $x \in V$, $y \in U$, $U \cap V = \phi$ and thus X is $Q^g - H_1$.

Theorem 2.19: Every $Q^g - normal$ space is $Q^g - U_1$.
Proof: Let X is $Q^g - normal$ space. Let x, y \in X such that $Q^g - cl \{x\} \cap Q^g - cl \{y\} = \phi$. Since X is $Q^g - normal$, there exists a $Q^g - open$ set U and a $Q^g - open$ set V such that $Q^g - cl \{x\} \subseteq V$, $Q^g -
cl \{y\} \subseteq U, Q^g - cl(U) \cap Q^g - cl(V) = \emptyset. \text{ Hence } x \in V, y \in U, Q^g - cl(U) \cap Q^g - cl(V) = \emptyset. \text{ Hence, } X \text{ is } Q^g - U_1.

REFERENCES