Parametric Design and Optimization of Cylindrical Gear

Xiao Liu¹, Xueyi Li²

¹²College of Mechanical and Electronic Engineering, Shandong University of Science and Technology, Qianwangang Road, Qingdao, Shandong, China

Abstract—Cylindrical gears have a wide range of applications in the manufacturing industry because of their advantages. And it is of great significance to study the design techniques and optimization methods of cylindrical gears. In this paper, parametric primary design of single-stage helical gear drive is discussed. And parametric design of the cylindrical gear is carried out by using optimized design method in the modern design method, the design variables, the objective function and the constraint condition are determined, and a complete mathematical model is established. Finally, a design example is given. On the MATLAB software platform, the mathematical model is solved by optimization toolbox. In case of meeting use requirement, using optimized design method to design cylindrical gears can reduce their volumes. However, the method proposed requires the initial design of the gear, and the calculation process is somewhat complicated.

Keywords—cylindrical gear; parametric design; strength check; optimized design

I. INTRODUCTION

Gear transmission is one of the most important transmission forms in mechanical transmission. It has the advantages of high efficiency, compact structure, long working life and stable transmission ratio. And the most widely used gear is involute gear drive. Types and specifications of gears are required by different kinds of machines and equipments, so the design and production of gears are still custom-made, and the design efficiency is low. For the design of cylindrical gear drive, the traditional design methods mainly rely on analysis, trial or analog to determine its complex structural parameters, thus reducing the viable design, so that the design becomes relatively passive. Designers often need to repeat to get more satisfactory results. With the continuous application of 3D CAD technology in the field of machinery, cylindrical gear design is developed to the integration, automation, intelligent direction. The parallelization of cylindrical gear design, analysis and manufacturing process shortens the cycle of cylindrical gear design and manufacturing. But at present, the design process of cylindrical gears still follows the traditional manual calculation method, thus affecting the overall efficiency of cylindrical gear design [¹]. For the safety of the design, the traditional cylindrical gear design will use a larger safety factor, resulting in redundancies of cylindrical gear performance and material, and can’t maximize the benefits. In the design, some parameters selected are not accurate enough and some parameters are interrelated. Sometimes in order to simplify calculation, some parameters will be taken as fixed values, which resulting in the design results are not accurate enough to meet the requirements of contemporary industry on the accuracy of cylindrical gears.

Similar to standard parts, cylindrical gear design follows a unified standard and unified algorithm to facilitate processing, so the design of cylindrical gear design method has universal significance.

Yu Zhiyong et al. put forward the parametric design method for straight gear with modification based on Pro/Program [²]. Wang Yingzi et al. studied the data processing method and the system construction problem in parametric design of gear transmission [³]. Taking improving the tooth face life as the goal, Yang Xiaoan optimized gears [⁴]. Shi Xiangkun studied the application of improved
genetic algorithm in gearbox optimization design and realized the three-dimensional parametric agile design of gear according to the optimization result [5]. Zhang Wei et al. explored the relationship between the magnitude of the gear displacement coefficient and size of the gears [6]. Luo Fei analyzed the characteristics of chart data in traditional design and put forward the programming method of data, table and line graph in gear transmission design [7].

Aiming at optimization problem of multistage cylindrical gear reducer, Thompson et al. proposed a multi-objective trade-off optimization method to minimize volume and improve tooth surface fatigue life. Rao et al. used four different hybrid optimization methods to study the constrained and unconstrained optimization problems of four stage cylindrical gear transmissions. Antal took the slip rate as the optimum target, and realized the allocation of the deflection coefficient of the helical gear drive in MATLAB [8]. Houser and other people established the mathematical model of the overall optimization problem [9] by taking the tooth shape parameter and the displacement coefficient as the optimization variables, and combining the structural optimization and the optimization of the displacement coefficients. The KISSsoft gear design software developed by KISSSoft, Switzerland, is a worldwide professional software tool for gear design, gear transmission system design and shaft and bearing design. The Romax Designer software developed by British Romax company is mainly used for the design and analysis of the virtual prototype of gear drive system [10]. In addition, the British SMT company's MASTA software and Dontyne System's Gear Production Suite software both have parametric design functions [11].

II. PARAMETRIC PRELIMINARY DESIGN OF CYLINDRICAL GEAR DRIVE

The preliminary design of the cylindrical gear drive, based on the input power $P$, the driving gear speed $n_1$, the gear ratio $u$, the working condition, the working life $N$ and so on, calculate the teeth number of drive gears $z_1$ and the number of driven gear $z_2$, modulus $m$, The pressure angle $\alpha$, the helix angle $\beta$, the drive wheel displacement coefficient $x_1$ and the driven wheel displacement coefficient $x_2$, the center distance $a$, the driving wheel tooth width $b_1$ and the driven wheel tooth width $b_2$ and select the material of the drive gear and the material of the driven gear and the accuracy level of the gear. The process shown in Figure 1:
III. OPTIMIZATION DESIGN BASED ON MATLAB

The optimal design of the machine is to determine the parameter variables of the problem and constrain the range of these variables, to determine the target of solving problems and to select a suitable optimization method solution under given conditions, environments and load conditions. Cylindrical gear is a widely used mechanical transmission device. It is an important way to optimize the design of cylindrical gear and select the optimum parameters for improving carrying capacity, reducing weights and costs of mechanical equipments.

3.1 Objective function
The volume of the cylindrical gear reducer is taken as the optimization target, that is to say, the structure is the most compact and the weight is lightest. The center distance of the reducer and the volume of the cylindrical gear are minimized. Therefore, the center distance \( a \) is chosen as the objective function. The expression of the center distance is:

\[
a = \frac{z_1 m (u + 1)}{2 \cos \beta} = \frac{x_1 x_2 (x_4 + 1)}{2 \cos x_3}
\]

Taking \( a_{\text{min}} \) as objective function:

\[
\min f(y) = \frac{x_1 x_2 (x_4 + 1)}{2 \cos x_3}
\]

3.2 Design variables
Independent parameters involved in the calculation of center distance include normal modulus \( m_n \), gear teeth number \( z_1 \), helix angle \( \beta \), and tooth number ratio \( u \). And the design variable is:

\[
X = [x_1, x_2, x_3, x_4]^T = [z_1, m, \beta, u]^T
\]
3.3 Constraints
In the structural optimization of an involute cylindrical gear reducer, the constraints considered include strength conditions, size conditions, lubrication conditions, and the limiting conditions for each optimization variable. In this paper, we discuss the single stage helical gear transmission, and there will be no interference of gear and shaft. Therefore, it does not need to consider size constraint and lubrication condition, only need to consider strength constraint and side constraint.

Based on above considerations, each constraint condition of single stage helical gear drive can be established in detail.

(1) Calculated contact stresses of small gear and big gear should be less than the allowable contact stress \( \sigma_{Hlim} \).

\[
g_1(X) = \frac{Z_B Z_H Z_v Z_e Z_\beta 2000 T_1 \cos^3 x_3 K_A K_v K_{H\beta} K_{H\alpha}}{x_1^3 \phi_d L_1} - \frac{\sigma_{Hlim} Z_{NT1} Z_{NT2} Z_{v1} Z_{W1} Z_{y1}}{S_{Hmin}} \leq 0
\]  

\[
g_2(X) = \frac{Z_D Z_H Z_v Z_e Z_\beta 2000 T_2 \cos^3 x_3 K_A K_v K_{H\beta} K_{H\alpha}}{x_1^3 \phi_d L_1} - \frac{\sigma_{Hlim} Z_{NT1} Z_{NT2} Z_{v2} Z_{W2} Z_{y2}}{S_{Hmin}} \leq 0
\]

In the formula, \( Z_B \) and \( Z_D \) are meshing coefficients of single gear pairs of small gears and large gears; \( \sigma_{Hlim} \) is contact fatigue limit of small (large) gear; \( Z_{NT1} \) (\( Z_{NT2} \)) is life factor for the calculation of contact fatigue of small (large) gear; \( Z_{L1} \) (\( Z_{L2} \)) is lubricant coefficient of small (large) gear; \( Z_{V1} \) (\( Z_{V2} \)) is speed coefficient of a small (large) gear; \( Z_{R1} \) (\( Z_{R2} \)) is roughness coefficient of a small (large) gear; \( Z_{W1} \) (\( Z_{W2} \)) is working hardening coefficient of small (large) gear teeth face; \( Z_{y1} \) (\( Z_{y2} \)) is size coefficient of the contact strength of small (large) gear; \( S_{Hmin} \) is minimum safety factor of gear contact strength \([13]\).

(2) The calculated bending stresses of small gear and big gear should be less than allowable bending stress.

\[
g_3(X) = \frac{2000 T_1 \cos^2 x_1 y_1 y_2 y_{F1} y_{S1} y_{\beta} y_{\alpha} y_{K_1} K_1 K_{F\beta} K_{F\alpha}}{x_1^2 \phi_d L_1} - \frac{\sigma_{Flim} Y_{ST1} Y_{NT1} Y_{Relt1} Y_{Relf1} y_{y1}}{S_{Fmin}} \leq 0
\]

\[
g_4(X) = \frac{2000 T_2 \cos^2 x_1 y_1 y_2 y_{F2} y_{S2} y_{\beta} y_{\alpha} y_{K_1} K_1 K_{F\beta} K_{F\alpha}}{x_1^2 \phi_d L_1} - \frac{\sigma_{Flim} Y_{ST2} Y_{NT2} Y_{Relt2} Y_{Relf2} y_{y2}}{S_{Fmin}} \leq 0
\]

\( Y_{F1} \) (\( Y_{F2} \)) is tooth profile coefficient of small (large) gear load acting on the outside of the single tooth meshing region; \( Y_{S1} \) (\( Y_{S2} \)) is stress correction factor of the small (large) gear load acting on the outside of the single tooth meshing zone; \( \sigma_{Flim} \) (\( \sigma_{Flim} \)) is bending fatigue limit of the root of a small (large) gear; \( Y_{ST1} \) (\( Y_{ST2} \)) is stress correction factor of a small (large) gear; \( Y_{NT1} \) (\( Y_{NT2} \)) is coefficient of
life calculated in terms of flexural fatigue strength; $Y_{\text{relT1}}$ ($Y_{\text{relT2}}$) is sensitive coefficient of the relative root fillet of small (large) gear; $Y_{\text{relT1}}$ ($Y_{\text{relT2}}$) is relative root surface condition coefficient of small (large) gear; $Y_{\text{y1}}$ ($Y_{\text{y2}}$) is size coefficient of a small (large) gear calculated in terms of bending strength; $S_{\text{Fmin}}$ is the minimum safety factor for the bending strength of the tooth root \cite{14}.

(3) Side constraints
1) Ensure that the small gear does not undercut.

$$g_3(X) = x_1 - \frac{2h_u^*}{\sin^2\left(\arctan\left(\frac{\tan\alpha_n}{\cos x_3}\right)\right)} \leq 0$$ (8)

$\alpha_n$ is the normal surface pressure angle of gear indexing circle, which is generally taken as $20^\circ$.

2) In order to limit the volume of gear pairs, the maximum number of pinion teeth is limited to 40.

$$g_6(X) = x_1 - 40 \leq 0$$ (9)

3) In order to limit the volume of the gear pair, the modulus are taken between 2~10.

$$g_7(X) = 2 - x_2 \leq 0$$ (10)

$$g_8(X) = x_2 - 10 \leq 0$$ (11)

4) In order to control the excessive axial thrust, the screw angle is limited to 8 degrees to 15 degrees.

$$g_9(X) = \frac{8\pi}{180} - x_3 \leq 0$$ (12)

$$g_{10}(X) = x_3 - \frac{15\pi}{180} \leq 0$$ (13)

Now, all the constraints are established, and a complete mathematical model can be formed:

$$X = [x_1, x_2, x_3, x_4]^T$$

$$\min f_i(Y) = \frac{x_1x_2(x_4 + 1)}{2\cos x_3}$$

$$s.t. \quad g_i(X) \leq 0 \quad i = 1, 2, 11, 10$$ (14)

(4) Optimization design based on MATLAB
Opening MATLAB software first of all, M file can be prepared. And then entering call file command in the MATLAB command window, optimized design parameters can be calculated based on the optimization toolbox in MATLAB.

IV. DESIGN EXAMPLE
We need to design a single-stage helical gear reducer used in a belt conveyor, input power $P = 10$KW, pinion speed $n_1 = 960$ r/min, gear ratio $u = 3.2$, driven by motor, working life of 15 years (annual work 300 days), two shifts, belt conveyor work smoothly, turn the same.

4.1 Preliminary design and strength check
According to the preliminary design method discussed in Section 2, calculated design results are: tooth number $z_1 = 29$, $z_2 = 93$, modulus $m_n = 2$mm, pressure angle $\alpha = 20^\circ$, helix angle $\beta = 14^\circ$. 
center distance $a = 125$ mm, tooth width $b_1 = 65$ mm, $b_2 = 60$ mm. The material of pinion is 40Cr (quenched and tempered), and the large gear is 45 steel (quenched and tempered). The gear is designed according to the 7-level accuracy.

Check the fatigue strength of tooth surface contact:

$$\sigma_H = \sqrt{\frac{2K_H T_1}{\phi_d d_1^3} \cdot \frac{u+1}{u} Z_H Z_k Z_e Z_\beta} = 485MPa < [\sigma_H] = 523MPa$$

Check the bending fatigue strength of tooth root:

Small gear: $\sigma_{F1} = \frac{2K_F T_1 Y_{F1} Y_{a1} Y_{Y} \cos^2 \beta}{\phi_d m_n^3 \varphi_1^2} = 124MPa < [\sigma_F]_1$

Big gear: $\sigma_{F2} = \frac{2K_F T_1 Y_{F2} Y_{a2} Y_{Y} \cos^2 \beta}{\phi_d m_n^3 \varphi_1^2} = 117MPa < [\sigma_F]_2$

Therefore, initial design results for this pair of gears meets the strength requirements.

![Figure 2. Preliminary design results of gear pair](image)

### 4.2 Optimal design

1. The transmission ratio of gear pair is given in this example, so the optimization variable can be taken as:

$$X = [x_1, x_2, x_3] = [z_1, m, \beta]^T \quad (15)$$

2. The minimum center distance is used as the objective function

$$\min f(X) = a = \frac{z_1 m(u+1)}{2 \cos \beta} = \frac{2.1 \cdot x_1 x_3}{\cos x_3} \quad (16)$$

3. Considering that the contact strength of the large gear is low, we only need to ensure the contact strength of the large gear, so $g_1(Y)$ is discarded, and other constraints are calculated as:
\[ g_2(Y) = 217356.4 \sqrt{\frac{\cos^3 y_3}{y_1^3 y_2^3}} - 523 \leq 0 \] (17)

\[ g_3(Y) = \frac{63.461 \times 10^4 \cos^2 y_3}{y_1^2 y_2^3} - 303.57 \leq 0 \] (18)

\[ g_4(Y) = \frac{53.188 \times 10^4 \cos^2 y_3}{y_1^2 y_2^3} - 238.86 \leq 0 \] (19)

\[ g_5(Y) = y_1 - \frac{2}{\sin^3 \left( \arctan \frac{\tan 20^\circ}{\cos y_3} \right)} \leq 0 \] (20)

\[ g_6(Y) = y_1 - 40 \leq 0 \] (21)

\[ g_7(Y) = 2 - y_2 \leq 0 \] (22)

\[ g_8(Y) = y_2 - 10 \leq 0 \] (23)

\[ g_9(Y) = \frac{8\pi}{180} - y_3 = 0.14 - y_3 \leq 0 \] (24)

\[ g_{10}(Y) = y_3 - \frac{15\pi}{180} = y_3 - 0.262 \leq 0 \] (25)

Therefore, the mathematical model is:

\[ X = [x_1, x_2, x_3]^T \]

\[ \min \quad f_i(Y) = \frac{2.1 \times x_1 x_2}{\cos x_3} \]

\[ s.t. \quad g_i(X) \leq 0 \quad i = 1, 2, 1, 10 \] (26)

(4) Solve the optimal solution of the optimization problem with the fmincon function in MATLAB [15].

Thus we can find the optimization results \( X = [27.56, 2.12, 13.82]^T \). Because the gear teeth number is integer, so \( z_1 = 28, z_2 = 89, m_n = 2, \beta = 13.82 \). And diameters of gears are smaller than preliminary design results.
V. CONCLUSION

In this paper, the preliminary design process of cylindrical gears was discussed firstly. In order to express the process clearly, the flow chart of the preliminary design of the cylindrical gear is given. Then, in order to minimize the cylindrical volume of the preliminary design, we selected the center distance of the cylindrical gear as the objective function, established the mathematical model of cylindrical gear optimization, and used MATLAB software to solve it. Finally, a design example was presented. Based on the optimum design method, the parameters of a set of cylindrical gears are optimized by using the MATLAB software. Compared to the preliminary design results, it can be found that the volumes of gears are minimized.

However, the calculation of this paper does not consider the displacement, but also confined to the simplest cylindrical gear. In future study, we will consider the displacement coefficient and bevel gears and other gears.

ACKNOWLEDGEMENT

This work was supported by the National Natural Science Foundation of China (Grant No. 51375282 & 51674155), the Special Funds for Cultivation of Taishan Scholars, the Shandong Provincial Natural Science Foundation of China (Grant No. ZR2015EM017), and the Science and Technology Development Program of Shandong Province (Grant No. 2014GGX103043).

REFERENCES


XIII. GBT 3480-1997 Calculation method of bearing capacity of involute cylindrical gears.