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Solution of Fuzzy Nonlinear Programming Problem Using Ranking Function

Dr.C. Loganathan[1] and M. Kiruthiga[2]
[1,2]Maharaja arts and science college,Coimbatore

Abstract : In this paper, we focus on a kind of nonlinear programming with fuzzy numbers and Ranking Functions. The coefficient of objective function and also right-hand side of the equation have fuzzy numbers. Then, we find the solution of the nonlinear programming problem by using Ranking Function, and we have given six numerical illustrative examples of the same.
***Keywords*:** Fuzzy nonlinearprogramming, Fuzzynumbers, Ranking functions,Trapezoidal fuzzy numbers.

I. INTRODUCTION

Ranking fuzzy numbers are an important tool in decision making. Making decisions involving multiple objectives is a daily task for a lot of people in the more diverse fields, and hence multiple objective decision making problems have defined a very well studied topic in the general area of decision making theory.

Decision making in fuzzy environments was introduced by Bellman and Zadeh (1970)[1]. Zimmerann proposed the concept of fuzzy linear programming[12].[8]Maleki proposed fuzzy variables in linear programming problems and introduced a new method for solving these problems using ranking functions.Tony Shaocheng, Buckile, Negi among others considered the situation where all parameters are in fuzzy[11].Hashem proposed the decision maker in the form of non symmetrical trapezoidal fuzzy numbers and get solution by ranking function[4].Pandian and jayalakmi[10],singh[7] proposed solving integer linear programming problems with fuzzy variables,fully fuzzy linearprogramming problems by using ranking function. P.Durga Prasad Dash solved the fuzzy multi objective non linear programming problem and fuzzy fracetional programming problems by using ranking function[2,3].

In this paper, we find the fuzzy nonlinear programming problems. First we take, when the co-efficients of objective functions are fuzzy numbers and right hand side are fuzzy numbers, too and both the co-efficients of objective function as well as right hand side are fuzzy numbers by using Maleki Ranking function when $\alpha = \beta$ and $\alpha \neq \beta$. A numerical examples is given to illustrate procedure.

II. PRELIMINARIES

Let Ã be a fuzzy number ie., a convex normalized fuzzy subset of the real line in the sense that:

Definition 2.1

$\exists x_0 \in R$ and $\mu_{\tilde{A}}(x_0) = 1$, where $\mu_{\tilde{A}}(x)$is the membership function specifying to what degree x belongs to Ã. $\mu_{\tilde{A}}$ is a piecewise continuous functions.

Definition 2.2

A fuzzy set Ã is called normal if its core is non-empty. In other words, there is at least one point $x \in X$ with $\mu_{\tilde{A}}(x) = 1$.

Definition 2.3

The α level set Ã is the set $\tilde{A}_\alpha = \{x \in R \mid \mu_{\tilde{A}}(x) \geq \alpha\}$ where α[0, 1]. The lower and upper bounds of any α level set $\tilde{A}_\alpha$ represented by $\inf_{x\in\tilde{A}_\alpha}$ and $\sup_{x\in\tilde{A}_\alpha}$ and we suppose that both are finite.

Definition 2.4

A function, usually denoted by 'L' or 'R', is a reference function of a fuzzy number iff

(1) $L(x) = L(-x)$

(2) $L(0) = 1$

(3) L is non increasing on [0, +∞].

(4)

Definition 2.5

A fuzzy set Ã on R is convex if for any x,y ∈ x and any α ∈ [0, 1] then
$\mu_{\tilde{A}}[\alpha x + (1-\alpha) y] \geq \min[\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)]$

Remark: A fuzzy set is convex if and only if all its α - cuts are convex.

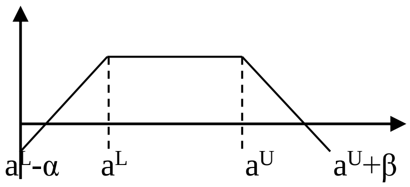
III. TRAPEZOIDAL FUZZY NUMBER:

There are various types of fuzzy numbers, but the triangular and trapezoidal are the most important fuzzy memberships. In this research we use the trapezoidal fuzzy numbers.

$$\mu_{\tilde{A}}(x) = \begin{cases} L((A^L - x) \mid \alpha) & \text{if } x \leq A^L, \alpha > 0 \\ R((x - A^U) \mid \beta) & \text{if } x \geq A^U, \beta > 0 \\ 1 & \text{otherwise,} \end{cases}$$

where $A^L < A^U$, $[A^L, A^U]$ is the core of Ã, $\mu_{\tilde{A}}(x) = 1$ ∀x + $[A^L, A^U]$, A^L, A^U are the lower and upper model values of Ã and α > 0, β > 0 are the left hand and right hand spreads [14].

More briefly, a flat fuzzy number is denoted by $\tilde{A} = [A^L, A^U, \alpha, \beta]_{LR}$



A trapezoidal fuzzy number can be show by $\tilde{a} = (a^L, a^U, \alpha, \beta)$, the support of ã is $(a^L - \alpha, a^U + \beta)$, and the core of ã is$[a^L, a^U]$, where F(R) denoted the set of all trapezoidal fuzzy numbers.

Let $\tilde{a} = (a^L, a^U, \alpha, \beta)$, $\tilde{b} = (b^L, b^U, \gamma, \theta)$ both the trapezoidal fuzzy numbers some of the results of applying fuzzy arithmetic on the fuzzy number ã and $\tilde{b}$ follow

Scalar Multiplication:

$x > 0, x \in R : x(.)\tilde{a} = (xa^L, xa^U, x\alpha, x\beta)$

$x < 0, x \in R : x(.)\tilde{a} = (xa^U, xa^L, -x\beta, -x\alpha)$

Addition:

$\tilde{a} + \tilde{b} = (a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \theta)$

Subtraction:

$\tilde{a} - \tilde{b} = (a^L - b^L, a^U - b^L, \alpha + \theta, \beta + \gamma)$

Let $\tilde{a}, \tilde{b}$ be fuzzy numbers and $S_L(\tilde{a}, \tilde{b}), S_R(\tilde{a}, \tilde{b})$ be the area determined by their membership functions according to the formula.

$$S_L(\tilde{a}, \tilde{b}) = \int_{I(\tilde{a},\tilde{b})} (\inf \tilde{a}_\alpha - \inf \tilde{b}_\alpha) d\alpha,$$

where $I(\tilde{a}, \tilde{b}) = \{\alpha \mid \inf \tilde{a}_\alpha \geq \inf \tilde{b}_\alpha\}$,is a sub set of $[\varepsilon, 1], \varepsilon > 0$ and

$$S_R(\tilde{a}, \tilde{b}) = \int_{S(\tilde{a},\tilde{b})} (\sup \tilde{a}_\alpha - \sup \tilde{b}_\alpha) d\alpha,$$

where $S(\tilde{a},\tilde{b})=\{\alpha \mid \sup \tilde{a}_\alpha \geq \sup \tilde{b}_\alpha\}$, is a subset of $[\varepsilon,1], \varepsilon>0$.

According to Roubens [5] the degree to which $\tilde{a}\geq\tilde{b}$ is defined as

$$C(\tilde{a},\tilde{b})=S_L(\tilde{a},\tilde{b})-S_L(\tilde{b},\tilde{a})+S_R(\tilde{a},\tilde{b})-S_R(\tilde{b},\tilde{a}).$$

We will consider that $\tilde{a}\geq\tilde{b}$ when $C(\tilde{a},\tilde{b})\geq 0$. According to the above definition Roubens proved the following proposition [9].

Proposition 3.1. Let $\tilde{a}$ and $\tilde{b}$ be *L – R fuzzy numbers with parameters* (a^L, a^U, α, β),(b^L, b^U, γ, θ) *and reference functions* $(L_{\tilde{a}},R_{\tilde{a}})$, $(L_{\tilde{b}},R_{\tilde{b}})$, *where all reference functions are invertible. Then* $\tilde{a}\geq\tilde{b}$ *if and only if*

$$\sup \tilde{a}_{\alpha_{\tilde{a},R}}+\inf \tilde{a}_{\alpha_{\tilde{a},L}} \geq \sup \tilde{b}_{\alpha_{\tilde{b},R}}+\inf \tilde{b}_{\alpha_{\tilde{b},L}},$$

where, if $k = \tilde{a}, \tilde{b}$

$$\alpha_{k,R}=R_k\left(\int_0^1 R_k^{-1}(\alpha)d\alpha\right),\ \alpha_{k,L}=L_k\left(\int_0^1 L_k^{-1}(\alpha)d\alpha\right)$$

As an illustration $\int_0^1 R_k^{-1}(\alpha)d\alpha$ is equal to:

$\frac{1}{2}$, if $R(\alpha)=1-\alpha, \alpha\in[0,1]$,

$\frac{2}{3}$, if $R(\alpha)=1-\alpha^2, \alpha\in[0,1]$,

$\frac{1}{3}$, if $R(\alpha)=1-\sqrt{\alpha}, \alpha\in[0,1]$,

$1-c^{-1}$, if $R(\alpha)=1-\alpha, \alpha\in[0,1]$,

In the case of trapezoidal fuzzy numbers, we have

$$\tilde{a}\geq\tilde{b} \text{ iff } a^L+a^U+\tfrac{1}{2}(\beta-\alpha)\geq b^L+b^U+\tfrac{1}{2}(\theta-\gamma). \quad (1)$$

IV. A FUZZY NUMBER NONLINER PROGRAMMING PROBLEM

Notation. We shall say that the real number f corresponds to the trapezoidal fuzzy number $\tilde{f}=(f^L,f^U,\alpha,\beta)$ if $f=f^L+f^U+\frac{1}{2}(\beta-\alpha)$.

Definition 4.1. Let F(R) be the set of all trapezoidal fuzzy numbers. The model

$$\begin{aligned}
\text{max:}\quad & \sum_{j=1}^{p}\tilde{C}_{rj}x_j^{\alpha j} && r = 1,2,3,\ldots q\\
\text{s.t}\quad & \sum_{j=1}^{p}\tilde{a}_{ij}x_j\leq\tilde{b}_i, && i = 1,2,\ldots,m_0, \qquad (2)\\
& \sum_{j=1}^{p}\tilde{a}_{ij}x_j\geq\tilde{b}_i, && i = m_0+1,\ldots,m,\\
& x_j\geq 0, && j = 1,2,\ldots p,
\end{aligned}$$

where $\tilde{a}_{ij}=(a_{ij}^L,a_{ij}^U,\alpha_{ij},\beta_{ij})$, $\tilde{b}_i=(b_i^L,b_i^U,\alpha_i,\beta_i)$ and $\tilde{c}_j=(c_j^L,c_j^U,\omega_j,\eta_j)\in F(R)$, $i = 1,2,\ldots,m$, $j=1,2,\ldots p$, is called a *fuzzy number non-linear programming model.*

Definition 4.2. Any set of $x_j^{\alpha j}\in x$ which satisfies the set of constraints (2) is called a feasible solution for (2). Let S be the set of all feasible solutions of (2). We shall say that $X^0\in S$ is an optimal feasible solution for (2) if $\tilde{C}X^0\geq\tilde{C}X$ for all $X\in S$.

Lemma 3.1 shows that we can reduce problem (2) to a problem in the classical form.

Lemma 4.1. *Problem (2) and the following problem are equivalent:*

max: $z = \sum_{j=1}^{p} c_j x_j^{\alpha j}$ r = 1,2,3,…q

s.t $\sum_{j=1}^{p} a_{ij} x_j \le b_i,$ $i = 1,2,...,m_0,$ (3)

$\sum_{j=1}^{p} a_{ij} x_j \ge b_i,$ $i = m_0 + 1,...,m,$

$x_j \ge 0,$ $j = 1,2,...p.$

Proof. Let S_1 and S_2 be the set of all feasible solutions of (2) and (3), respectively.

Then X ∈ S_1 if and only if:

$$\sum_{j=1}^{p} \tilde{a}_{ij} x_j \le b_i, \quad i = 1,2,...,m_0, \quad \sum_{j=1}^{p} \tilde{a}_{ij} x_j \ge b_i, \quad i = m_0 + 1,...,m,$$

if and only if:

$$\sum_{j=1}^{p} \{a_{ij}^L, a_{ij}^U, \alpha_{ij}, \beta_{ij}\} x_j \le \{b_i^L, b_i^U, \alpha_i, \beta_i\}, \quad i = 1,2,....m_0,$$

$$\sum_{j=1}^{p} \{a_{ij}^L, a_{ij}^U, \alpha_{ij}, \beta_{ij}\} x_j \ge \{b_i^L, b_i^U, \alpha_i, \beta_i\}, \quad i = m_0 + 1,....m,$$

if and only if:

$$\left\{ \sum_{j=1}^{p} x_j a_{ij}^L, \sum_{j=1}^{p} x_j a_{ij}^U, \sum_{j=1}^{p} x_j \alpha_{ij}, \sum_{j=1}^{p} x_j \beta_{ij} \right\} \le \{b_i^L, b_i^U, \alpha_i, \beta_i\}, \quad i = 1,2,....m_0,$$

$$\left\{ \sum_{j=1}^{p} x_j a_{ij}^L, \sum_{j=1}^{p} x_j a_{ij}^U, \sum_{j=1}^{p} x_j \alpha_{ij}, \sum_{j=1}^{p} x_j \beta_{ij} \right\} \ge \{b_i^L, b_i^U, \alpha_i, \beta_i\}, \quad i = m_0 + 1,....m,$$

if and only if:

$$\sum_{j=1}^{p} \{a_{ij}^L + a_{ij}^U + (\beta_{ij} - \alpha_{ij})/2\} x_j \le \{b_i^L + b_i^U + (\beta_i - \alpha_i)\}, \quad i = 1,2,....m_0,$$

$$\sum_{j=1}^{p} \{a_{ij}^L + a_{ij}^U + (\beta_{ij} - \alpha_{ij})/2\} x_j \ge \{b_i^L + b_i^U + (\beta_i - \alpha_i)\}, \quad i = m_0 + 1,....m,$$

if and only if:

$$\sum_{j=1}^{p} a_{ij} x_j \le b_i, \quad i = 1,2,...,m_0, \quad \sum_{j=1}^{p} a_{ij} x_j \ge b_i, \quad i = m_0 + 1,...,m,$$

if and only if:

$X \in S_2.$

Hence $S_1 = S_2$.

Now suppose that X^0 is an optimal feasible solution for (2), then for all X ∈ S_1 we have:

$\tilde{C} X^0 \ge \tilde{C} X$

if and only if:

$$\sum_{j=1}^{p} \tilde{c}_j x_j^{\alpha j^0} \ge \sum_{j=1}^{p} \tilde{c}_j x_j^{\alpha j}$$

if and only if:

$$\sum_{j=1}^{p} \{c_j^L + c_j^U + (\eta_j - \omega_j)/2\} x_j^{\alpha j^0} \ge \sum_{j=1}^{p} \{c_j^L + c_j^U + (\eta_j - \omega_j)/2\} x_j^{\alpha j}$$

if and only if:

$$\sum_{j=1}^{p} c_j x_j^{\alpha j^0} \geq \sum_{j=1}^{p} c_j x_j^{\alpha j}$$

We conclude that X^0 is an optimal feasible solution for (3).

The ranking function is approach of ordering fuzzy numbers which is an efficient. Ranking function have been introduced which are used for solving non linear programming problem with fuzzy parameters. The ranking function is denoted by F(R) →F(R) is the set of fuzzy numbers defined on real line, where a natural order exist.

Suppose ã and $\tilde{b}$ be two trapezoidal fuzzy numbers, then the ranking function of F(R) is as following

If $\tilde{a} \geq \tilde{b}$ then $R(\tilde{a}) \geq R(\tilde{b})$
If $\tilde{a} > \tilde{b}$ then $R(\tilde{a}) > R(\tilde{b})$
If $\tilde{a} = \tilde{b}$ then $R(\tilde{a}) = R(\tilde{b})$

Where ã and $\tilde{b}$ are in F(R), also in the same way we can write $\tilde{a} \leq \tilde{b}$ iff $\tilde{b} \geq \tilde{a}$.

Lemma 4.2: Let R be any Ranking function, then;

1- $\tilde{a} \geq \tilde{b}$ iff $\tilde{a} - \tilde{b} \geq 0$ iff $-\tilde{b} \geq -\tilde{a}$
2- $\tilde{a} \geq \tilde{b}$ and $\tilde{c} \geq \tilde{d}$, then $\tilde{a} + \tilde{c} \geq \tilde{b} + \tilde{a}$

In this research, we use the form of linear ranking function, such that $R(K\tilde{a} + \tilde{b}) = KR(\tilde{a}) + R(\tilde{b})$, where $K \in R$.

Maleki ranking function: [8]

Let $\tilde{a} = (a^L, a^U, \alpha, \beta)$ be a fuzzy numbers, then the ranking function is,

$$R(\tilde{a}) = \int_0^1 (\inf \tilde{a}_\alpha + \sup \tilde{a}_\alpha) d\alpha$$

$$R(\tilde{a}) = a^L + a^U + \frac{1}{2}(\beta - \alpha) \qquad (4)$$

Where $\alpha = \beta$ or $\alpha \neq \beta$.

V. FUZZY NON LINEAR PROGRAMMING PROBLEM:

The crisp nonlinear programming problem defined as follows:

$$\text{Max (or) min } z = \sum_{j=1}^{n} \tilde{c}_j x_j^{\alpha j}$$

$$\text{s.t} \quad \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \qquad i = 1,2,\ldots,m, \qquad (5)$$

$$x_j \geq 0$$

where $c_j \in R^n$, $b_i \in R^m$, $a_{ij} \in R^{n \times m}$

The parameters in the above model are crisp. But if some or all parameters are fuzzy number then the crisp became fuzzy nonlinear programming model.

Algorithm:

Let us discuss the three stages of fuzzy non linear programming problem[6].

Step -1

In this stage, we make the objective function coefficients as trapezoidal fuzzy numbers which is as follows:

$$\max \ z = \sum_{j=1}^{n} \tilde{c}_j x_j^{\alpha j}$$

$$\text{s.t} \quad \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \qquad i = 1,2,\ldots,m, \qquad (6)$$

where c_j – fuzzy coefficients of objective function, $b_i \in R^m$, $a_{ij} \in R^{n \times m}$ then we solve the fuzzy non linear programming by Maleki Ranking function

$$\max\ z = \sum_{j=1}^{n} [c_j^L + c_j^U + \tfrac{1}{2}(\beta - \alpha)] x_j^{\alpha j}$$

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i, \qquad i = 1,2,\ldots,m, \qquad (7)$$

$$x_j \ge 0$$

Step -2

In this stage, we make the right-hand side coefficients as trapezoidal fuzzy numbers which is as follows:

$$\max\ z = \sum_{j=1}^{n} c_j x_j^{\alpha j}$$

$$\text{s.t} \quad \sum_{j=1}^{n} a_{ij} x_j \le \tilde{b}_i, \qquad i = 1,2,\ldots,m, \qquad (8)$$

$$x_j \ge 0$$

where b_i – fuzzy right-hand side coefficients, $c_j \in R^n$, $a_{ij} \in R^{n \times m}$ then we solve the fuzzy non linear programming by Maleki Ranking function

$$\max\ z = \sum_{j=1}^{n} c_j x_j^{\alpha j}$$

$$\sum_{j=1}^{n} a_{ij} x_j \le [b_i^L + b_i^U + \tfrac{1}{2}(\beta - \alpha)], \qquad i = 1,2,\ldots,m, \qquad (9)$$

$$x_j \ge 0$$

Step -3

In this stage, we make both the objective function coefficients and right-hand side coefficients as trapezoidal fuzzy numbers which is as follows:

$$\max\ z = \sum_{j=1}^{n} \tilde{c}_j x_j^{\alpha j}$$

$$\text{s.t} \quad \sum_{j=1}^{n} a_{ij} x_j \le \tilde{b}_i, \qquad i = 1,2,\ldots,m, \qquad (10)$$

$$x_j \ge 0$$

where b_i and c_j are trapezoidal fuzzy numbers, $a_{ij} \in R^{n \times m}$ then we solve the fuzzy non linear programming by Maleki Ranking function

$$\max\ z = \sum_{j=1}^{n} [c_j^L + c_j^U + \tfrac{1}{2}(\beta - \alpha)] x_j^{\alpha j}$$

$$\text{s.t} \quad \sum_{j=1}^{n} a_{ij} x_j \le [b_i^L + b_i^U + \tfrac{1}{2}(\beta - \alpha)], \qquad i = 1,2,\ldots,m, \qquad (11)$$

$$x_j \ge 0$$

VI. NUMERICAL EXAMPLES:

In this section we explain all stages by the following examples which is suggest by the researchers in case of $\boldsymbol{\alpha = \beta}$ or $\boldsymbol{\alpha \neq \beta}$.

Example: 1

Step: 1

The following examples suggested by the researchers in case $\alpha = \beta$.

$$\max\ \tilde{z} = (4,8,2,2)\tilde{x}_1 + (1,3,1,1)\tilde{x}_2^2 + (1,5,2,2)\tilde{x}_3$$

$$\text{s.t:} \quad 2\tilde{x}_1 - \tilde{x}_2 + 2\tilde{x}_3 \le 4$$

$\tilde{x}_1 + 4\tilde{x}_3 \le 4$

$\tilde{x}_1 + 3\tilde{x}_2 + 2\tilde{x}_3 \le 7$ (12)

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \ge 0$

Solution: By using Maleki ranking function we get

max $z = 12x_1 + 4x_2^2 + 6x_3$

s.t $2x_1 - x_2 + 2x_3 \le 4$

$x_1 + 4x_3 \le 4$

$x_1 + 3x_2 + 2x_3 \le 7$ (13)

$x_1, x_2, x_3 \ge 0$

We solve the above crisp non linear programming problem we get the following solution

$x_1 = 2.2, x_2 = 1.9, x_3 = 0, z = 40.3$

Step: 2

max $z = 6x_1 + 2x_2^2 + 3x_3$

s.t $2x_1 - x_2 + 2x_3 \le (1,7,3,3)$

$x_1 + 4x_3 \le (2,6,2,2)$

$x_1 + 3x_2 + 2x_3 \le (5,9,2,2)$ (14)

$x_1, x_2, x_3 \ge 0$

Solution:

max $z = 6x_1 + 2x_2^2 + 3x_3$

s.t $2x_1 - x_2 + 2x_3 \le 8$

$x_1 + 4x_3 \le 8$

$x_1 + 3x_2 + 2x_3 \le 14$ (15)

$x_1, x_2, x_3 \ge 0$

The solution for nonlinear programming (15) is as follows

$x_1 = 5.43, x_2 = 3.86, x_3 = 0, z = 40.32$

Step: 3

max $\tilde{z} = (4,8,2,2)\tilde{x}_1 + (1,3,1,1)\tilde{x}_2^2 + (1,5,2,2)\tilde{x}_3$

s.t: $2\tilde{x}_1 - \tilde{x}_2 + 2\tilde{x}_3 \le (1,7,3,3)$

$\tilde{x}_1 + 4\tilde{x}_3 \le (2,6,2,2)$

$\tilde{x}_1 + 3\tilde{x}_2 + 2\tilde{x}_3 \le (5,9,2,2)$ (16)

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \ge 0$

Solution:

max $z = 12x_1 + 4x_2^2 + 6x_3$

s.t $2x_1 - x_2 + 2x_3 \le 8$

$x_1 + 4x_3 \le 8$

$x_1 + 3x_2 + 2x_3 \le 14$ (17)

$x_1, x_2, x_3 \ge 0$

Then the solution for non linear programming (17) is as follows

$x_1 = 5.321, x_2 = 3.86, x_3 = 0, z = 52.33$

Example: 2

Step: 1

In the following example is suggested by researchers in case $\alpha \neq \beta$.

$$\max\ \tilde{z} = (4,8,3,1)\tilde{x}_1 + (1,3,1,1)\tilde{x}_2^2 + (1,5,3,1)\tilde{x}_3$$

s.t:
$$2\tilde{x}_1 - \tilde{x}_2 + 2\tilde{x}_3 \leq 4$$
$$\tilde{x}_1 + 4\tilde{x}_3 \leq 4$$
$$\tilde{x}_1 + 3\tilde{x}_2 + 2\tilde{x}_3 \leq 7 \qquad (18)$$
$$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq 0$$

Solution: By using Maleki ranking function we get

$$\max\ z = 11x_1 + 4x_2^2 + 5x_3$$

s.t
$$2x_1 - x_2 + 2x_3 \leq 4$$
$$x_1 + 4x_3 \leq 4$$
$$x_1 + 3x_2 + 2x_3 \leq 7 \qquad (19)$$
$$x_1, x_2, x_3 \geq 0$$

We solve the above crisp non linear programming problem we get the following solution

$$x_1 = 3.714, x_2 = 2.428, x_3 = 0, z = 38.571$$

Step: 2

$$\max\ z = 6x_1 + 2x_2^2 + 3x_3$$

s.t
$$2x_1 - x_2 + 2x_3 \leq (1,7,4,2)$$
$$x_1 + 4x_3 \leq (2,6,1,3) \qquad (20)$$
$$x_1 + 3x_2 + 2x_3 \leq (5,9,1,3)$$
$$x_1, x_2, x_3 \geq 0$$

Solution:

$$\max\ z = 6x_1 + 2x_2^2 + 3x_3$$

s.t
$$2x_1 - x_2 + 2x_3 \leq 7 \qquad (21)$$
$$x_1 + 4x_3 \leq 9$$
$$x_1 + 3x_2 + 2x_3 \leq 15$$
$$x_1, x_2, x_3 \geq 0$$

Then the solution for non linear programming (21)

$$x_1 = 5.143, x_2 = 4.286, x_3 = 0, z = 38.429$$

Step: 3

$$\max\ \tilde{z} = (4,8,3,1)\tilde{x}_1 + (1,3,1,1)\tilde{x}_2 + (1,5,3,1)\tilde{x}_3$$

s.t:
$$2\tilde{x}_1 - \tilde{x}_2 + 2\tilde{x}_3 \leq (1,7,4,2)$$
$$\tilde{x}_1 + 4\tilde{x}_3 \leq (2,6,1,3)$$
$$\tilde{x}_1 + 3\tilde{x}_2 + 2\tilde{x}_3 \leq (5,9,1,3)$$
$$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq 0 \qquad (22)$$

Solution: by using Maleki function we get

$$\max\ z = 11x_1 + 4x_2 + 5x_3$$

s.t
$$2x_1 - x_2 + 2x_3 \leq 7$$
$$x_1 + 4x_3 \leq 9$$
$$x_1 + 3x_2 + 2x_3 \leq 15$$
$$x_1, x_2, x_3 \geq 0$$

VII. CONCLUSION

The co-efficients of objective function and right hand side with fuzzy numbers with special ranking function for Maleki linear ranking function. For all six stage which solving in the paper, noting that,
$x_3 = 0$, but x_1 and x_2 are not. For the three states with $\alpha = \beta$ and $\alpha \neq \beta$.

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