A comparative on Study of delaminated plate and shells

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Abstract: Fiber reinforced composite plates and shells are progressively supplanting conventional metallic ones. The assembling procedure and administration of the composite overlays often prompt delamination. Delamination lessens the firmness and quality of composite covers since they permit out of plane relocation of handles to happen all the more effectively. Dynamic soundness investigation is an indispensable piece of most building structures. The present work manages the investigation of the impacts of free vibration, clasping and dynamic strength of delaminated cross utilize composite plates and shells. A first request shear twisting hypothesis in light of limited component model is created for examining the unsteadiness locale of mid plane delaminated composite plate and shell.

The essential comprehension of the impact of the delamination on the characteristic frequencies, non-dimensional clasping load and non-dimensional excitation recurrence of composite plates and shells is exhibited. Moreover, different elements influencing the vibration, clasping and dynamic shakiness district of delaminated composite plates and shells are talked about. The numerical results for the free vibration, clasping and dynamic soundness of covered cross-utilize plates and shells with delamination are exhibited. Not surprisingly, the regular frequencies and the basic clasping heap of the plates and shells diminish with expansion in delamination. Increment in delamination likewise causes dynamic precariousness locales to be moved to lower excitation frequencies.

I. INTRODUCTION

Composite laminates are broadly utilized as a part of building structures because of their great properties, for example, high quality to-weight proportion, high stiffness-to-weight proportion and outline flexibility and so forth. Delamination can bring about genuine auxiliary corruption. Which is a debonding or separation between individual handles of the cover, often happens in composite laminates. Delamination may emerge amid assembling (e.g., deficient wetting, air entanglement) or amid administration (e.g., low speed sway, fledgling strikes). They may not be obvious or scarcely unmistakable at first glance, since they are installed inside the composite structures. Be that as it may, the nearness of delamination may fundamentally lessens the stiffness and quality of the structures and may influence some configuration parameters, for example, the vibration normal for structure of structure. (e.g., regular recurrence and mode shape). Delaminations decrease the common recurrence, as an immediate consequence of the diminishment of stiffness, which may bring about reverberation if the lessened recurrence is near the working recurrence. It is hence essential to comprehend the execution of delaminated composites in a dynamic situation. The subject of foreseeing the dynamic and mechanical conduct of delaminated structures has along these lines pulled in impressive consideration.

Importance of the stability studies of delaminated composite shell

- Delamination between plies is one of the most common defects encountered in composite laminates. Delamination can cause serious structural degradation. Which is a debonding or separation between individual plies of the laminate, frequently occurs in composite laminates.
- Structural elements subjected to in-plane periodic forces may lead to parametric resonance, due to certain combinations of the values of load parameters. The instability may occur below the critical load of the structure under compressive loads over wide ranges of excitation frequencies.
II. MATHEMATICAL FORMULATION

Governing equation for analysis:

Equation for free vibration is,

\[ ([K] - \omega^2 [M]) \{ \phi \} = \{ 0 \} \]

Equation for buckling analysis is,

\[ ([K] - P [K_a]) \{ \phi \} = \{ 0 \} \]

Equation for dynamic stability analysis in line,

\[ K + (\alpha_0 \pm 0.5 \alpha_1) P_c K_c - 0.25 M \theta^2 = 0 \]

III. FINITE ELEMENT ANALYSIS

A laminated doubly curved shell panel of length \( a \), width \( b \) and thickness \( h \) consisting of \( n \) arbitrary number of anisotropic layers is considered. The displacement field is related to mid-plane displacements and rotations as

\[ u(x,y,z,t) = u^0(x,y,z,t) + z \theta_x(x,y,t), \]
\[ v(x,y,z,t) = v^0(x,y,z,t) + z \theta_y(x,y,t), \]
\[ w(x,y,z,t) = w^0(x,y,z,t), \]

Using Sander’s first approximation theory for thin shells, the generalized strains in terms of mid-plane strains and curvatures are expressed as

\[ \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yy} & \gamma_{xy} & \gamma_{xz} & \gamma_{yz} \end{bmatrix}^T = \begin{bmatrix} \varepsilon_{0xx}^0 & \varepsilon_{0yy}^0 & \gamma_{0xy}^0 & \gamma_{0xz}^0 & \gamma_{0yz}^0 \end{bmatrix}^T + \begin{bmatrix} k_{xx} & k_{yy} & k_{xy} & k_{xz} & k_{yz} \end{bmatrix}^T \]

Laminated doubly curved composite shell axes

The stress resultants are related to the mid-plane strains and curvatures for a general laminated shell element as
Where $N_x$, $N_y$ and $N_{xy}$ are in-plane stress resultants, $M_x$, $M_y$ and $M_{xy}$ are moment resultants and $Q_x$, $Q_y$ are transverse shear stress resultants. The extensional, bending-stretching and bending stiffness’s of the laminate are expressed in the usual form as

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{z=1}^{z=2} a_i(z) dz, \quad i,j = 1,2,6.$$

Similarly, the shear stiffness is expressed as

$$(A_{ij}) = \sum_{k=1}^{n} \int_{z=1}^{z=2} a_k(z) dz, \quad i,j = 4,5.$$

The element stiffness matrix, $[K_e]$ is given by

$$[K_e] = \int_{-1}^{1} \int_{-1}^{1} [B]^T [D] [B] J d\xi d\eta$$

Similarly the consistent element mass matrix $[M_e]$ is expressed as

$$[M_e] = \int_{-1}^{1} \int_{-1}^{1} [N]^T [\rho] [N] J d\xi d\eta$$

The shape functions $N_i$ are defined as

$$N_i = (1+\xi+i)(1-1)/4 \quad i=1to4$$

$$N_i = (1-\xi^2)(1+1)/2 \quad i=5,7$$

$$N_i = (1+\xi^2)(1-1)/2 \quad i=6,8$$

**Strain displacement Relations**

The linear strains are defined as
Assuming that \( w \) does not vary with \( z \), the non-linear strains of the shell are expressed as

\[
\varepsilon_{xi} = \frac{\partial u}{\partial x} + \frac{w}{R_x} + Zk_x
\]

\[
\varepsilon_{yi} = \frac{\partial u}{\partial y} + \frac{w}{R_y} + Zk_y
\]

\[
\gamma_{xyi} = \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x} + \frac{2w}{R_{xy}} + Zk_{xy}
\]

\[
\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} - C_1 \frac{u}{R_x} - C_1 \frac{v}{R_{xy}}
\]

\[
\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} - C_1 \frac{u}{R_y} - C_1 \frac{v}{R_{xy}}
\]

The linear strain can be described in term of displacements as

\[
\{\varepsilon\} = [B] \{\delta e\}
\]

Where \( \{\delta e\} = [u_1 \ v_1 \ w_1 \ \theta_{x1} \ \theta_{y1} \ u_2 \ v_2 \ \ldots \ u_8 \ v_8 \ w_8 \ \theta_{x8} \ \theta_{y8}]^T \)

Generalised element mass matrix or consistent mass matrix:

\[
[M_g] = \int \int [N]^T[p][N] J dxdy
\]

IV. GEOMETRIC STIFFNESS MATRIX:
The element geometric stiffness matrix is derived using the non-linear in-plane Green’s strains. The strain energy due to initial stresses is

\[
U_2 = \int \int [\alpha]^T \{\varepsilon_{ni}\} dV
\]

Using non-linear strains, the strain energy can be written in matrix form as
The strain energy becomes

\[ U_2 = \frac{1}{2} \int \{ \delta_e \}^T [\mathbf{S}] [\mathbf{G}] \{ \delta_e \} dV = \frac{1}{2} \{ \delta_e \}^T [\mathbf{K}_g] \{ \delta_e \} \]

Where element geometric stiffness matrix

\[ [\mathbf{K}_g] = \int \int [\mathbf{G}]^T [\mathbf{S}] [\mathbf{G}] dV \]

\[
\begin{bmatrix}
N_{i\alpha} & 0 & 0 & 0 & 0 \\
N_{i\beta} & 0 & 0 & 0 & 0 \\
0 & N_{i\alpha} & 0 & 0 & 0 \\
0 & N_{i\beta} & 0 & 0 & 0 \\
-N_i/R_x & 0 & N_{i\alpha} & 0 & 0 \\
0 & -N_i/R_y & N_{i\beta} & 0 & 0 \\
0 & 0 & 0 & N_{i\alpha} & 0 \\
0 & 0 & 0 & N_{i\beta} & 0 \\
0 & 0 & 0 & 0 & N_{i\alpha} \\
0 & 0 & 0 & 0 & N_{i\beta}
\end{bmatrix}
\]

V. ANALYSIS METHOD

The finite element formulation is developed for the dynamic analysis of laminated composite shells with delamination using the first order shear deformation theory. An eight-noded continuous doubly curved isoparametric element is employed in the present analysis with five degrees of freedom viz. \( u, v, w, \theta_x, \) and \( \theta_y \) at each node.

The eigenvalue equation for the free vibration analysis of laminated composite plate and shell can be expressed as

\[ ([\mathbf{K}] - \omega^2 [\mathbf{M}]) \{ \phi \} = \{ 0 \}, \]

The eigenvalue equation for the stability analysis of laminated composite plate and shell can be expressed as

\[ ([\mathbf{K}] - \mathbf{P}[\mathbf{K}_g]) \{ \phi \} = \{ 0 \} \]

The eigenvalue equation for the dynamic stability analysis of laminated composite plate and shell can be expressed as

\[ K + (\alpha_0 \pm 0.5\alpha_1) P_{cr} K_G - 0.25M\theta^2 = 0 \]
Where M, K and KG are the mass, the stiffness and the geometric matrices. \( \theta \) represent the frequency.
\( \alpha_0 \) and \( \alpha_1 \) are static and dynamic parameters taking values from 0 to 1.

VI. RESULTS

Boundary conditions:
- Simply supported boundary
  \( v=w=\dot{y}=0 \) at \( x=0, a \) and \( u=w=\dot{x}=0 \) at \( y=0 \).
- Clamped boundary
  \( u=v=w=\dot{x}=\dot{y}=0 \) at \( x=0, a \) and \( y=0 \).
- Free edges
  No restrain

Non-dimensionalisation of Parameters:

<table>
<thead>
<tr>
<th>No</th>
<th>Parameter</th>
<th>Composite plates/shells</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Frequency of vibration ( (\omega) )</td>
<td>( \omega_a^2 \sqrt{\rho/h^2 E_z} )</td>
</tr>
<tr>
<td>2</td>
<td>Buckling load ( (\lambda) )</td>
<td>( N_x \frac{a^2}{E_z h^3} )</td>
</tr>
<tr>
<td>3</td>
<td>Frequency of excitation ( (\Omega) )</td>
<td>( \frac{\Omega}{\omega} \sqrt{\rho/h^2 E_z} )</td>
</tr>
</tbody>
</table>

Vibration of composite plates and shells

Geometry and material properties: \( a=b=0.5m, \ a/h=100, \ R/a=5 \) and \( R/a=10, \ \rho = 1600kg/m^3 \).

\[ E_{11} = 172.5GPa, \ E_{22} = 6.9GPa, \ G_{12} = G_{13} = 3.45GPa, \ G_{23} = 1.38GPa. \]

<table>
<thead>
<tr>
<th>% delamination</th>
<th>Stacking sequence</th>
<th>Spherical shell ((R/a=10))</th>
<th>Cylindrical shell ((R/a=10))</th>
<th>Plate((R/a=\infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>present</td>
<td>Parhi et</td>
<td>present</td>
</tr>
<tr>
<td>0</td>
<td>(0/90)_2</td>
<td>129.1353</td>
<td>129.20</td>
<td>103.0197</td>
</tr>
<tr>
<td>25</td>
<td>(0/90)_2</td>
<td>104.5625</td>
<td>104.59</td>
<td>69.5945</td>
</tr>
<tr>
<td>56.25</td>
<td>(0/90)_2</td>
<td>98.3438</td>
<td>98.36</td>
<td>59.9258</td>
</tr>
</tbody>
</table>

Natural frequencies (Hz) for mid-plane delaminated simply supported composite
spherical and cylindrical shells with different delamination.

<table>
<thead>
<tr>
<th>% delamination</th>
<th>Stacking sequence</th>
<th>Spherical shell (R/a=5)</th>
<th>Cylindrical shell (R/a=5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>present</td>
<td>Parhi et al</td>
<td>present</td>
</tr>
<tr>
<td>0</td>
<td>(0/90)</td>
<td>201.8568</td>
<td>202.02</td>
</tr>
<tr>
<td>25</td>
<td>(0/90)</td>
<td>187.4469</td>
<td>187.51</td>
</tr>
<tr>
<td>56.25</td>
<td>(0/90)</td>
<td>183.9246</td>
<td>183.96</td>
</tr>
</tbody>
</table>

**Buckling of composite plates:**
Comparison of non-dimensional buckling loads of a square simply supported doubly curved panel with (0/90) lamination.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>R_x/a=5, R_y/a=5</td>
<td>12.018</td>
<td>11.920</td>
<td>12.214</td>
</tr>
<tr>
<td>R_x/a=10, R_y/a=5</td>
<td>11.649</td>
<td>11.515</td>
<td>11.822</td>
</tr>
<tr>
<td>R_x/a=10, R_y/a=20</td>
<td>11.245</td>
<td>11.250</td>
<td>11.479</td>
</tr>
<tr>
<td>R_x/a=20, R_y/a=20</td>
<td>11.187</td>
<td>11.164</td>
<td>11.406</td>
</tr>
<tr>
<td>Plate</td>
<td>11.114</td>
<td>11.115</td>
<td>11.353</td>
</tr>
</tbody>
</table>

Comparison of non-dimensional buckling loads of a square simply supported symmetric cross-ply cylindrical shell panels with [0/90/0/90/0] lamination for different length-to-thickness ratio (a/h).

**Numerical Results:**

**Numerical results of vibration**
Table shows Natural frequencies (Hz) for 25% delaminated cross ply-(0/90)_{n} simply supported composite spherical and cylindrical shells with different no. of layers

<table>
<thead>
<tr>
<th>No. of layers</th>
<th>Spherical shell (R/a=5)</th>
<th>Cylindrical shell (R/a=5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>188.8401</td>
<td>107.0587</td>
</tr>
<tr>
<td>4</td>
<td>187.4469</td>
<td>104.5625</td>
</tr>
<tr>
<td>8</td>
<td>190.7261</td>
<td>110.4770</td>
</tr>
<tr>
<td>16</td>
<td>191.5230</td>
<td>111.8759</td>
</tr>
</tbody>
</table>
Table shows Natural frequencies (Hz) for 25% delaminated cross ply-(0/90)_2 simply supported composite spherical and cylindrical shells with different aspect ratio.

<table>
<thead>
<tr>
<th>a/b</th>
<th>Spherical shell</th>
<th>Cylindrical shell</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R_x/a=5, R_y/b=5</td>
<td>R_x/a=10, R_y/b=5</td>
</tr>
<tr>
<td>0.5</td>
<td>293.8187</td>
<td>220.6784</td>
</tr>
<tr>
<td>1</td>
<td>187.4469</td>
<td>104.5625</td>
</tr>
<tr>
<td>1.5</td>
<td>153.3563</td>
<td>71.5290</td>
</tr>
<tr>
<td>2</td>
<td>134.7433</td>
<td>56.4072</td>
</tr>
</tbody>
</table>

**25% delamination**

![Graph showing natural frequencies for different aspect ratios of spherical and cylindrical shells.]

**25% delamination on cross ply**

![Graph showing natural frequencies for different aspect ratios of spherical and cylindrical shells with delamination.]
VII. CONCLUSION

Vibration study

- The effects of dynamic behaviour on delaminated composite plates and shells under free vibration conclude that for particular % of delamination, the natural frequencies increase with increase of number of layers due to effect of bending-stretching coupling.
- With increase of aspect ratio, the natural frequency decreases.
- With increase of % delamination, the natural frequency decreases and it also observed the frequency of vibration increase with decrease of b/h ratio of cross ply panels with delamination. This is due to reduction in stiffness caused by delamination.

Dynamic stability study

- The onset of instability occurs earlier with increase in percentage of delamination.
- It also observed that with increases of number of layers the excitation frequency increases. This is due to increase of stiffness caused by bending-stretching coupling with increase of layers.
- The dynamic instability region occurs earlier with decrease of degree of orthotropy and due to decrease of delamination the onset instability region shifted to lower frequency to higher frequency and also the width of instability regions increased with decrease of delamination.

REFERENCES