APPLICATIONS OF MANPOWER WITH VARIOUS STAGES IN BUSINESS USING STOCHASTIC MODELS

Dr. R. Arumugam¹, M. Rajathi²
¹Assistant Professor, Department of Mathematics, Periyar Maniammai University, Thanjavur-613 403.
²Assistant Professor, Swami Vivekanda College, Thanjavur

Abstract: The present study aim is to find the steady rate of crisis and steady state of probabilities under different conditions which are manpower, under irregular conditions of nil, moderate and full availability in the case of business and manpower. The different states have been discussed under the different assumption that the transition from one state to another in both business and manpower arise in exponential time with different parameters.

Keywords: Markov chain, Steady state, Crisis rate.

I. INTRODUCTION

Nowadays we establish that labor has become a buyers’ market as well as sellers’ market. Any business which usually runs on commercial base wishes to keep only the optimum level of all resources needed to meet company’s responsibility at any time during the course of the business and so manpower is not an exemption. This is spelt in the sense that a company may not want to keep manpower more than what is needed. Hence, retrenchment and recruitment are general and recurrent in most of the companies now. Recruitment is done when the business is busy and shed manpower when the business is lean. Equally true with the labor, has the choice to switch over to other jobs because of improved working condition, better emolument, nearness to their living place or other reasons. Under such conditions the company may face crisis because business may be there but skilled manpower may not be available. If skilled laborers and technically qualified persons leave the business the seriousness is most awfully felt and the company has to appoint paying deep price or pay overtime to employees.

Approach to manpower problems have been dealt in very many different ways as early as 1947 by Vajda [11] and others. Manpower planning models have been dealt in depth in Barthlomew [1], Grinold & Marshal [3] and Vajda[11]. The methods to calculate wastages (Resignation, dismissal and death) and promotion intensities which produce the proportions corresponding to some preferred planning proposals has been dealt by Lesson [4]. Markov model are designed for promotion and wastages in manpower system by Vassilou [12]. Subramaniam [11] in his thesis has made an attempt to give optimal policy for recruitment training, promotion, and wastages in man power planning models with special provisions such as time bound promotions, cost of training and voluntary retirement scheme. For an application of Markov chains in a manpower system with seniority and efficiency and Stochastic structures of graded size in manpower planning systems one may refer to Sethhare [9]. A two unit stand by system has been examined by Chandrasekar and Natrajnan [2] with confidence limits under steady state. For n unit standby system one may refer to Ramanarayanan and Usha [8]. Yadhavalli and Botha [13] have observed the same for two unit system with introduction of preparation time for the service facility and the confidence limits for stationary rate of disappointment of an erratically used system. For three characteristics system involving manpower, machine and money one may refer to C. Mohan and
R. Ramanarayanan [6]. For the study of Semi Markov Models for Manpower planning one may refer to the paper by Sally Meclean [5]. Stochastic Analysis of a Business with Varying Levels in Manpower and Business has been discussed by C. Mohan and P. Selvaraju [7].

**Markov chain model with eight point state space**

In this paper we consider two characteristics namely business and manpower. There are business concerns like business construction companies or any other industry which get full business when nominal staff is available or business may be roughly nil but staff strength may be full, one more possibility is that moderate business may be there during the period when manpower is nil, moderate and full. The steady state probabilities of the continuous Markov chain relating the transitions in different states are derived and critical states are identified for presenting the cost analysis. Numerical illustrations are also provided.

**II. ASSUMPTIONS**

1. There are three levels of manpower namely manpower is full, moderate and nil.
2. There are three levels of business namely, (a) business is fully available (b) business is moderate (c) business is nil.
3. The time T- during which the business remains continuously filled and becomes nil has exponential distribution with parameter $\alpha_{10}$ and the time R- required to stop full recruitment from zero stage is exponentially distributed with parameter $\beta_{01}$.
4. The fully available and zero periods of the manpower are exponentially distributed with parameters ‘$\lambda$’ and ‘$\mu$’ respectively.
5. While manpower becomes zero, the business is vanished and becomes nil.
6. The time $T'$ - during which the business remains continuously filled and becomes nil has an exponential distribution with parameter $\alpha_{20}$ and the time $R'$ - required to stop full recruitment from zero stage is exponentially distributed with parameter $\beta_{02}$.
7. The time $T''$ - during which the business is continuously full becomes moderate has an exponential distribution with parameter $\alpha_{31}$ and the time $R''$ - required to stop full recruitment from moderate level is exponentially distributed with parameter $\beta_{12}$.
8. The time $T'''$ - during which the business is moderately full becomes continuously filled has an exponential distribution with parameter $\alpha_{12}$ and the time $R'''$ - required to stop moderate recruitment from full recruitment is exponentially distributed with parameter $\beta_{31}$.
9. $T''''$, $R''''$, $T'''$ and $R'''$, $T'$ and $R'$; T- and R- are independently distributed random variables.

**III. SYSTEM ANALYSIS**

The Stochastic Process $X(t)$ describing the state of the system is Markov chain continuous time with four points state space as given below in the order of manpower and business

$$S = \{(0\ 0), (0\ 1), (0\ 2), (1\ 0), (1\ 1), (1\ 2), (2\ 0), (2\ 1)\}$$

Where,

- 0 – Refers to shortage/non availability of manpower and business. The system is in state (1, j).
- When the manpower is in state 1 and business is in state for (1, j) = (1, 1) or (1 0) or (0 0) and there is not the state (0 1) as the business is misplaced while the manpower goes off.
- 1 – Refers to moderate availability of business and it refers to full availability of manpower in the case of business.
2 – Refers to full availability in the case of business.

The continuous time Markov chain of the state space is given below which is a matrix of order eight.

\[
Q = \begin{bmatrix}
M/P/B & (0 0) & (0 1) & (0 2) & (1 0) & (1 1) & (1 2) & (2 0) & (2 1) \\
(0 0) & \gamma_1 & \beta_{01} & \beta_{02} & \mu & 0 & 0 & 0 & \beta_{21} \\
(0 1) & \alpha_{10} & \gamma_2 & \beta_{12} & 0 & \mu & 0 & \beta_{21} & 0 \\
(0 2) & \alpha_{20} & \alpha_{21} & \gamma_3 & 0 & 0 & \mu & 0 & \beta_{01} \\
(1 0) & \lambda & 0 & 0 & \gamma_4 & \beta_{01} & \beta_{02} & \beta_{12} & 0 \\
(1 1) & 0 & \lambda & 0 & \alpha_{10} & \gamma_5 & \beta_{12} & 0 & \beta_{21} \\
(1 2) & 0 & 0 & \lambda & \alpha_{20} & \alpha_{21} & \gamma_6 & \beta_{10} & 0 \\
(2 0) & 0 & \alpha_{12} & 0 & \alpha_{10} & \gamma_7 & \beta_{10} & \gamma_8 & 0 \\
(2 1) & \alpha_{12} & 0 & \alpha_{10} & 0 & \alpha_{21} & 0 & \alpha_{01} & \gamma_8 \\
\end{bmatrix}
\]

Where, \(\gamma_1 = (\beta_{01} + \beta_{02} + \mu + \beta_{21}); \quad \gamma_2 = (\alpha_{10} + \beta_{12} + \mu + \beta_{21})\)

\(\gamma_3 = (\alpha_{20} + \alpha_{21} + \mu + \beta_{01}); \quad \gamma_4 = (\lambda + \beta_{01} + \beta_{02} + \beta_{12})\)

\(\gamma_5 = (\lambda + \alpha_{10} + \beta_{12} + \beta_{21}); \quad \gamma_6 = (\lambda + \alpha_{20} + \alpha_{21} + \beta_{10})\)

\(\gamma_7 = -(\alpha_{12} + \alpha_{21} + \alpha_{10} + \beta_{10}); \quad \gamma_8 = -(\lambda + \alpha_{10} + \alpha_{21} + \alpha_{01})\)

The occurrences of transition in both business and manpower are independent, the individual infinitesimal generator of them are given by

Let \(\pi = [\pi_{21}, \pi_{20}, \pi_{11}, \pi_{10}, \pi_{01}, \pi_{02}, \pi_{00}]\) be the steady state probability vector of the matrix \(Q\), then

\[\pi Q = 0, \quad \pi e = 1\]

Using (2), we get the steady state probabilities:

(i) The infinitesimal generator of manpower of order two is as follows

\[
M = \begin{bmatrix}
M/P & 1 & 0 \\
0 & \lambda & -\lambda \\
0 & \mu & -\mu \\
\end{bmatrix}
\]

The steady state probabilities are \(\pi_{M1} = \frac{\mu}{\mu + \lambda}\) and \(\pi_{M0} = \frac{\lambda}{\mu + \lambda}\)

(ii) The infinitesimal generator of business is given by the matrix of order three.

\[
M = \begin{bmatrix}
M/P & 0 & 1 & 2 \\
0 & -(\beta_{02} + \beta_{01}) & \beta_{01} & \beta_{02} \\
1 & \alpha_{10} & -(\alpha_{12} + \alpha_{10}) & \alpha_{12} \\
2 & \alpha_{20} & \alpha_{21} & -(\alpha_{20} + \alpha_{21}) \\
\end{bmatrix}
\]

The steady state probabilities of manpower are

\[\pi_{M2} = \frac{d_2}{d_0 + d_1 + d_2}; \quad \pi_{M1} = \frac{d_1}{d_0 + d_1 + d_2}; \quad \pi_{M0} = \frac{d_0}{d_0 + d_1 + d_2}\]

Where, \(d_0 = \beta_{12} \alpha_{20} + \alpha_{10} \alpha_{20} + \alpha_{10} \alpha_{21}\); \(d_1 = \beta_{01} \alpha_{20} + \beta_{01} \alpha_{21} + \beta_{02} \alpha_{21}\)

\(d_2 = \beta_{02} \alpha_{10} + \beta_{02} \beta_{12} + \beta_{01} \beta_{12}\)
The steady state probability vectors are \( \pi Q = 0 \) and \( \pi e = 1 \)

\[
\begin{align*}
\pi_{00} &= \frac{d_0 \lambda}{z \sum t_i d_i}; \\
\pi_{01} &= \frac{d_1 \lambda}{z \sum t_i d_i}; \\
\pi_{02} &= \frac{d_2 \mu}{z \sum t_i d_i}; \\
\pi_{10} &= \frac{d_0 \mu}{z \sum t_i d_i}; \\
\pi_{11} &= \frac{d_1 \mu}{z \sum t_i d_i}; \\
\pi_{12} &= \frac{d_2 \mu}{z \sum t_i d_i}; \\
\pi_{20} &= \frac{d_0 \beta_{21}}{z \sum t_i d_i}; \\
\pi_{21} &= \frac{d_1 \beta_{21}}{z \sum t_i d_i}; \\
\pi_{22} &= \frac{d_2 \beta_{21}}{z \sum t_i d_i}
\end{align*}
\]

where \( z \sum_{i=0}^{2} d_i = [d_0 + d_1 + d_2] \) and \( Z = [b + a] \).

While manpower is available business is fully available or nil. Manpower is inadequate or nil will lead to crisis state.

The only crisis states are \{ (1 1), (1 2), (2 1) \} and the crisis arise when there is full business or moderate business also the manpower is moderate or full. Now the rate of crisis in the steady state is given by

\[
C_{\infty} = \beta_{01} \pi_{10} + \beta_{12} \pi_{11} + \lambda_{10} \pi_{12} + \lambda_{01} \pi_{20}
\]

Using steady state probabilities, we obtain

\[
C_{\infty} = \frac{\lambda}{2 Z \sum_{i=0}^{2} t_i d_i} \left[ \mu (d_0 \beta_{01} + d_1 \beta_{12} + d_2 \lambda_{10}) + d_0 \beta_{21} \lambda_{01} \right] - \quad (5)
\]

**IV. NUMERICAL ILLUSTRATION AND STEADY STATE COST CALCULATION**

Now taking the values of the parameters in the model as below, we can find the steady state probabilities and the rate of crisis using (4) and (5).

\[\alpha_{10} = 6, \alpha_{20} = 4, \alpha_{21} = 5, \beta_{01} = 9, \beta_{02} = 6, \beta_{12} = 12, \beta_{21} = 8, \lambda = 13 \text{ and } \mu = 15\]

<table>
<thead>
<tr>
<th>Steady state probability</th>
<th>( \pi_{00} )</th>
<th>( \pi_{01} )</th>
<th>( \pi_{02} )</th>
<th>( \pi_{10} )</th>
<th>( \pi_{11} )</th>
<th>( \pi_{12} )</th>
<th>( \pi_{20} )</th>
<th>( \pi_{21} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.4884</td>
<td>0.2953</td>
<td>0.3978</td>
<td>0.1657</td>
<td>0.1460</td>
<td>0.2386</td>
<td>0.0518</td>
<td>0.0456</td>
</tr>
</tbody>
</table>

Now assigning the values of \( b=3, 5, 7, 9, 11, 13, 15 \text{ and } 17 \). We calculate the corresponding rate of crisis and it is tabulated below:

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>13</th>
<th>15</th>
<th>17</th>
<th>19</th>
<th>21</th>
<th>23</th>
<th>25</th>
<th>27</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{\infty} )</td>
<td>170.85</td>
<td>134.04</td>
<td>114.57</td>
<td>102.51</td>
<td>94.31</td>
<td>88.37</td>
<td>83.87</td>
<td>80.34</td>
</tr>
</tbody>
</table>
The steady state costs in various situations are determined taking the values:

\[ C^0_M = 45, \quad C^1_M = 35, \quad C^2_M = 38, \quad C^0_B = 54, \quad C^1_B = 35, \quad C^2_B = 42. \]

Steady state probability cost = \( \pi_{ij} \left( C^i_M + C^j_B \right) \) \( \lambda \) \( 10 \) \( \alpha \) \( 0 \) \( \beta \) \( \gamma \) \( \delta \) \( \epsilon \) \( \zeta \) \( \eta \) \( \theta \) \( \iota \) \( \kappa \) \( \lambda \) \( \mu \) \( \nu \) \( \xi \) \( \omicron \) \( \pi \) \( \rho \) \( \sigma \) \( \tau \) \( \upsilon \) \( \phi \) \( \chi \) \( \psi \) \( \omega \) \( \Gamma \) \( \Delta \) \( \Theta \) \( \Lambda \) \( \Xi \) \( \Pi \) \( \Sigma \) \( \Upsilon \) \( \Phi \) \( \Psi \) \( \Omega \)

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Steady state probability</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \pi_{00} )</td>
<td>48.350</td>
</tr>
<tr>
<td>2</td>
<td>( \pi_{01} )</td>
<td>23.624</td>
</tr>
<tr>
<td>3</td>
<td>( \pi_{02} )</td>
<td>34.608</td>
</tr>
<tr>
<td>4</td>
<td>( \pi_{10} )</td>
<td>14.747</td>
</tr>
<tr>
<td>5</td>
<td>( \pi_{11} )</td>
<td>10.220</td>
</tr>
<tr>
<td>6</td>
<td>( \pi_{12} )</td>
<td>18.377</td>
</tr>
<tr>
<td>7</td>
<td>( \pi_{20} )</td>
<td>4.766</td>
</tr>
<tr>
<td>8</td>
<td>( \pi_{21} )</td>
<td>3.329</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>158.021</td>
</tr>
</tbody>
</table>

**V. CONCLUSION**

It is found that as the value of \( \lambda \) increases and the corresponding crisis rate decreases. Also it is observed that the cost is high if there is full business but there is no manpower, under such situations labour has to be paid a lot. When the manpower is full, there is a chance of the business getting into crisis state if manpower leaves particularly experts and experienced people leave the concern. The same holds in the case of business are moderate while the manpower may be full or inadequate.
REFERENCES

2. Chandraswkar, Natrajan, Two unit stand by system with confidence limits under steady state (1997).