Mining Ore Transportation Cost Modelling

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Abstract: Aljalamid phosphate ore deposit, in the northern part of Saudi Arabia, is spread on three different locations that are Fish area, Southern area and Western area. Thus, determination of the optimum location of the mineral processing plant, to serve the three different locations with minimum transportation cost, represents a real challenge. Therefore, this paper aims to build a mathematical model to find the location of the intended processing plant. The adopted mathematical model takes into consideration number of locations, tonnage in each location, and the distance from the geometric center of each location to an unknown optimum location of the processing plant. To solve the considered mathematical model, FORTRAN program was developed. The coordinates of the optimum location of processing plant were obtained. The model is applicable to serve any other ore deposit especially that consists of many locations.

Keywords: Mathematical Modeling, Ore Transportation Cost, Minimization Cost, Processing Plant Location, Gravity Center Calculation.

I. INTRODUCTION

The ore transportation cost is one of the main important operating costs of any mining project. Transportation cost depends on a number of factors such as distance, quantity, and method of transportation. Many models were formulated such as Larwood and Benson Model [1], Anderson Model [2], Bechtel Model [3], and Zimmerman Model [4]. There were many trials to minimize the transportation cost [5-10]. Fjellstrom [11] determined transportation cost of ore and waste material to the crusher and backfilling rooms in the underground Renstrom mine by using the software package "AutoMod". AutoMod is a simulation program that can simulate all the processes not only in the mine but also in other industries. Main results from the model indicate that the mine truck is 23% more effective than the highway trucks, the usage of only highway trucks are 34% more expensive than the usage of only mine trucks and for combination alternatives, if most of the transportation is done by mine trucks instead of highway trucks, the transportation cost per ton decreases by 10-20%. Wegener [12] suggested recent developments in the field of transportation models. Brazil et al. [13] considered the case of optimizing the construction and transportation costs of underground mining roads. The authors focused on the model of underground mine networks. This model consisted of ramps and their relations with maximum gradient. They stated that the cost is affected by ramps lengths, the ore quantity transported through the ramps and their gradients.

Dharma and Ahmad [14] investigated two models for a real-world application of a transportation problem that involved transporting iron ore from two iron ore mines to three steel plants using both linear and integer programming methods. Their models were then further applied to generate an optimized solution that minimized the transportation cost. The authors compared their results to determine the most practical model for a real-world situation and how significant the difference would be.

Shephard [15] suggested a model based on the transportation cost factors such as quantity, distance, shipment delay, transport technology, and route. Inwood and Keay [16] used modern compiled evidence on effective transport costs of iron trade to investigate the relationship between trade costs...
and trade volumes. Reeb and Leavengood [17] used linear programming to minimize transportation cost. Ali and Sik [18] presented a method according to linear programming to minimize the transportation cost in mining. Chen et al. [19] proposed an organization optimization model based on the mathematical model of classical transportation problem and transport path. Optimization method for imported iron ore transportation from the perspective of integrated transportation was built which focused on the optimal transport spatial distribution and path for single freight flow in multi-transportation network without the demand matrix.

Ahmed et al. [20] used Linear Programming Problem to minimize the transportation cost. Joshi [21] optimized a technique to reduce transportation problem cost. Transportation problem was formulated as a linear programming problem and was solved by using four methods (northwest corner, least cost, Vogel, and Modi). Ahmad [22] presented a description for solution technique called Best Candidates Method (BCM) for solving optimization problems. His proposed technique aimed to get the optimal solution. The previous transportation models did not present any trials to determine the optimum processing plant location according to ore transportation cost minimization from different locations, Hence this study aims to develop a mathematical model to find an optimum location of the processing plant based on ore transportation cost.

II. THEORETICAL CONSIDERATION

2.1. General
Selection of optimum location of processing plant depends upon the total transportation cost. Transportation cost is dependent mainly upon the reserves of each ore deposit and the distances of ore transportation from each ore deposit to the suggested optimum location. To select the optimum location, the following mathematical model is suggested.

2.2. Mathematical model
The main idea of the suggested mathematical model depends on the minimum sum of weighted distances. This can be graphically illustrated as shown in Fig.1, which shows an ore deposit scattered into different locations (say n). Each location has an ore tonnage of \(Q_i\) which is considered to be concentrated at a single point that is the center of gravity having coordinates \(x_i, y_i, z_i\). All of the ore tonnages are to be transported to a location where the mineral processing plant is to be built so that the transportation cost should be minimized. Hence, this issue can be mathematically expressed as in equations from 1 to 6.

Fig. 1. Schematic diagram for optimum location of the processing plant.
According to Fig. 1, the main idea of the suggested mathematical model depends on the minimum sum of weighted distances as given in equation (1)

\[ \sum_{i=1}^{n} Q_i D_i = \text{Minimum} \]  

(1)

Where:

\( n \): Number of ore deposit locations.
\( Q_i \): Reserves of each ore deposit location.
\( D_i \): Distance between any ore deposit location and the processing plant optimum location, \( D_i \) can be mathematically expressed as \( \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} \).

\( x, y, z \): The coordinates of the gravity center of the processing plant optimum location.
\( x_i, y_i, z_i \): The coordinates of the gravity center of different ore deposit locations.

The sum of weighted distances is:

\[ S = \sum_{i=1}^{n} Q_i D_i \]  

(2)

Or

\[ S = \sum_{i=1}^{n} Q_i \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} \]  

(3)

For this sum to be minimum, the partial differentiation in regards to \( x, y \) and \( z \) should equal zero, which means that the following conditions have to be satisfied:

\[ \frac{\partial S}{\partial x} = \sum_{i=1}^{n} Q_i \frac{x-x_i}{\sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}} = 0 \]  

(4)

\[ \frac{\partial S}{\partial y} = \sum_{i=1}^{n} Q_i \frac{y-y_i}{\sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}} = 0 \]  

(5)

\[ \frac{\partial S}{\partial z} = \sum_{i=1}^{n} Q_i \frac{z-z_i}{\sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}} = 0 \]  

(6)

Where:

\( S \): The sum of weighted distances
\( D_i \): The distance from any gravity center of ore deposit to the processing plant optimum location.
\( \frac{\partial S}{\partial x}, \frac{\partial S}{\partial y}, \frac{\partial S}{\partial z} \): The partial differentials.
\( n \): The number of ore deposit locations.
\( Q_i \): The reserves of each ore deposits.
\( x, y, z \): The coordinates of optimum location of processing plant.
\( x_i, y_i, z_i \): The coordinates of the geometric center of different ore deposits.

2.3. Calculation of the center of gravity of ore deposit

The purpose of center of gravity calculation is to accumulate the reserves of each location at a certain point that has \( x_i, y_i \) and \( z_i \) coordinates.
The resultant of the weights, or parallel forces of gravity, on all the particles of a body always passes through a certain particle or point fixed with reference to the body is turned. This particle or point is the center of gravity of the body [23]. Calculation of the center of gravity of ore deposit can be calculated as in the following steps.

First Step: The ore deposit area is divided into triangles. Each triangle consists of three boreholes (Fig. 2).

![Fig. 2. The ore deposit area divided into triangles.](image)

Second Step: Calculation of the ore reserves and center of gravity of each triangle. The center of gravity of a triangle (Fig. 3) is the intersection point of its medians [24].

![Fig. 3. Determination of the center of gravity of a triangle.](image)

The center of gravity divides each of the medians in the ratio 2:1, which is to say it is located ⅓ of the perpendicular distance between each side and the opposing point. Its Cartesian coordinates are the means of the coordinates of the three vertices. That is, if the three vertices are \( a = (x_a, y_a) \), \( b = (x_b, y_b) \), and \( c = (x_c, y_c) \), then the x and y coordinates of the center of gravity \( \overline{x}, \overline{y} \) and can be calculated as shown in equation (7).

\[
C.G. = (\overline{x}, \overline{y}) = \frac{1}{3} (a + b + c) = \left[ \frac{1}{3} (X_a + X_b + X_c), \frac{1}{3} (Y_a + Y_b + Y_c) \right]
\] ..........................(7)

Third Step: Determination of Z coordinate of the center of gravity of a triangle. 
Z coordinate of gravity center of a triangle can be determined for the point, which was determined in the second step. Z coordinate of gravity center of the triangle can be calculated from equation (8).

\[
\overline{z} = z - t_{a,b} - \frac{t_b}{2}
\] ..........................(8)
Where:

- $z^*$: $z$ coordinate of the gravity center of the triangle.
- $z$: $z$ coordinate of ground surface.
- $t_{o,b}$: The thickness of overburden.
- $t_b$: The thickness of the bed of required block.

Fourth Step: Determination of the center of gravity of ore deposit that consists of a number of triangles (Fig. 4) by using the following equations (9,10,11).

\[
\begin{align*}
  x &= \frac{\sum_{i=1}^{n} R_i x_i}{\sum_{i=1}^{n} R_i} \quad \ldots \quad (9) \\
  y &= \frac{\sum_{i=1}^{n} R_i y_i}{\sum_{i=1}^{n} R_i} \quad \ldots \quad (10) \\
  z &= \frac{\sum_{i=1}^{n} R_i z_i}{\sum_{i=1}^{n} R_i} \quad \ldots \quad (11)
\end{align*}
\]

Where:

- $x$: $x$ coordinate of the center of gravity of the ore deposit.
- $y$: $y$ coordinate of the center of gravity of the ore deposit.
- $z$: $z$ coordinate of the center of gravity of the ore deposit.
- $x_i$: $x$ coordinate of the center of gravity of the $i^{th}$ triangle of ore deposit.
- $y_i$: $y$ coordinate of the center of gravity of the $i^{th}$ triangle of ore deposit.
- $z_i$: $z$ coordinate of the center of gravity of the $i^{th}$ triangle of ore deposit.
- $R_i$: The reserves of the $i^{th}$ triangle of ore deposit.
- $n$: Number of triangles of ore deposit.

III. RESULTS AND DISCUSSIONS

3.1. The center of gravity of ore deposit.

Data of 500 boreholes were obtained from Ma’aden Company for Aljalamid deposit. These boreholes represent the different three locations. Calculation of the coordinates of the gravity center of ore deposit consists of two steps. The first is calculation of the gravity center of each block.
(triangle) of ore deposit using equations (7,8). The second step is the application of the moment method (Equations 9,10,11). The ore reserves and gravity center of different locations of Aljalamid phosphate ore deposits were calculated and shown in Table 1, which illustrates that the Fish area has the highest reserve approaching 451 million tons.

Table 1. Reserves and coordinates of center of gravity of different locations of Aljalamid phosphate ore deposit.

<table>
<thead>
<tr>
<th>Location No.</th>
<th>Ore deposit location</th>
<th>Reserves (Tons)</th>
<th>Center of Gravity (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fish Area</td>
<td>450,765,474</td>
<td>x = 291095 y = 245985 z = 756</td>
</tr>
<tr>
<td>2</td>
<td>Southern Area</td>
<td>361,831,423</td>
<td>x = 290119 y = 191354 z = 712</td>
</tr>
<tr>
<td>3</td>
<td>Western Area</td>
<td>338,756,461</td>
<td>x = 242008 y = 216954 z = 723</td>
</tr>
</tbody>
</table>

3.2. Determination of Optimum Processing Plant Location

The optimum location of processing plant requires the number of ore deposit locations(n), tonnage ($Q_i$) and center of gravity for each location ($x_i,y_i,z_i$). The Equations (4, 5&6) of the mathematical model can be used in the form of 12,13 and 14 as shown in the following.

$$\frac{\partial s}{\partial x} = \frac{Q_1 * (x - x_1)}{\sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2}} + \frac{Q_2 * (x - x_2)}{\sqrt{(x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2}} + \cdots + \frac{Q_n * (x - x_n)}{\sqrt{(x-x_n)^2 + (y-y_n)^2 + (z-z_n)^2}} = 0 \cdots (12)$$

$$\frac{\partial s}{\partial y} = \frac{Q_1 * (y - y_1)}{\sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2}} + \frac{Q_2 * (y - y_2)}{\sqrt{(x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2}} + \cdots + \frac{Q_n * (y - y_n)}{\sqrt{(x-x_n)^2 + (y-y_n)^2 + (z-z_n)^2}} = 0 \cdots (13)$$

$$\frac{\partial s}{\partial z} = \frac{Q_1 * (z - z_1)}{\sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2}} + \frac{Q_2 * (z - z_2)}{\sqrt{(x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2}} + \cdots + \frac{Q_n * (z - z_n)}{\sqrt{(x-x_n)^2 + (y-y_n)^2 + (z-z_n)^2}} = 0 \cdots (14)$$

The unknowns in the equations 12, 13 and 14 are x, y & z (The coordinates of the optimum location of the processing plant). To solve these equations, FORTRAN program was developed. The flow chart of the FORTRAN program is shown in Fig. 5.
3.3. Model validation
The hitherto derived mathematical model depends on equations 12, 13 and 14 which are physically derived equations. Hence validation will be carried out according to the following steps:

1. Assume number of locations (n)
2. Assume tonnage for each location $Q_i$
3. Assume coordinates of C.G for each location $(x_i, y_i, z_i)$
4. Enter these assumptions in program
5. Run the program and get the output i.e processing plant optimum location coordinates $(x, y, z)$.
6. Substitute in equations (12, 13 and 14) using the optimum location coordinates $(x, y, z)$ resulted from step 5, and assumed values of $Q_i$ (step 2) and its corresponding C.G coordinates (step 3) and calculate the values of the equations.
7. The model will be valid if the calculated values in step 6 were zero or within the permissible errors
8. Steps from 1 to 7 are to be repeated for different cases with varied assumptions of the program inputs (variables), n, and Q, x, y, z for each location.

Table 2 shows the obtained results for 10 validation cases together with their corresponding errors. As an example, case no 1 is shown in Figs. 6 and 7. Where Fig. 6 shows a snapshot of the program with the assumed input variables and Fig. 7 shows the a snapshot of program final results of the optimum location.
Fig. 6. A snapshot of the program with case no.1 variables entered

<table>
<thead>
<tr>
<th>Location No.</th>
<th>Qi, Xi, Yi, Zi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10000, 1000, 1000, 100</td>
</tr>
<tr>
<td>2</td>
<td>15000, 3000, 2000, 200</td>
</tr>
<tr>
<td>3</td>
<td>20000, 2000, 3000, 300</td>
</tr>
</tbody>
</table>

Fig. 7. A snapshot of the program with optimum location coordinates for case no.1

<table>
<thead>
<tr>
<th>Optimum Location</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>X = 2034.29257</td>
<td>0.00000E+00</td>
</tr>
<tr>
<td>Y = 2895.39495</td>
<td>0.00000E+00</td>
</tr>
<tr>
<td>Z = 289.53949</td>
<td>0.00000E+00</td>
</tr>
</tbody>
</table>
Table 2. Validation Table.

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Location No.1</th>
<th>Location No.2</th>
<th>Location No.3</th>
<th>Location No.4</th>
<th>Optimum Location</th>
<th>Eq.</th>
<th>Eq.</th>
<th>Eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ton m m m m</td>
<td>ton m m m m</td>
<td>ton m m m m</td>
<td>ton m m m m</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 3</td>
<td>100 100 100 10 150 300 200 20 200 200 300 20</td>
<td></td>
<td></td>
<td></td>
<td>2034.2</td>
<td>2895.3</td>
<td>289.53</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9257</td>
<td>9495</td>
<td>949</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>2 4</td>
<td>500 500 500 50 800 100 150 70 700 200 100 60 900 250 500 800 0 0 0 0 0</td>
<td></td>
<td></td>
<td></td>
<td>1997.6</td>
<td>998.13</td>
<td>60.125</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2097</td>
<td>330</td>
<td>98</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>3 3</td>
<td>600 800 700 90 750 110 110 11 900 130 50 800 110 300 100 5 0 0 0 0 0</td>
<td></td>
<td></td>
<td></td>
<td>1126.6</td>
<td>885.40</td>
<td>109.49</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3943</td>
<td>152</td>
<td>180</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>4 4</td>
<td>750 120 100 15 800 150 180 14 900 200 160 17 680 250 900 13</td>
<td></td>
<td></td>
<td></td>
<td>1580.0</td>
<td>1368.0</td>
<td>143.46</td>
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</tr>
<tr>
<td>0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9806</td>
<td>2748</td>
<td>178</td>
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</tr>
<tr>
<td>5 3</td>
<td>150 294 250 11 165 335 390 11 173 386 282 12</td>
<td></td>
<td></td>
<td></td>
<td>35624.7</td>
<td>29567.6</td>
<td>1172.7</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>57185.0</td>
<td>31558</td>
<td>0062</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>6 4</td>
<td>258 153 112 27 195 172 133 24 188 181 128 26 210 191 109 23</td>
<td></td>
<td></td>
<td></td>
<td>17591.0</td>
<td>12530.0</td>
<td>256.78</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>21462.0</td>
<td>80223</td>
<td>175</td>
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</tr>
<tr>
<td>7 3</td>
<td>310 369 112 32 344 309 156 33 442 418 173 36</td>
<td></td>
<td></td>
<td></td>
<td>37298.0</td>
<td>14357.0</td>
<td>338.66</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>700 70 30 0 270 00 10 9 870 30 30 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>58255.0</td>
<td>96841</td>
<td>142</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>6 4</td>
<td>570 553 617 12 390 683 813 12 410 548 732 12 871 831 954 13</td>
<td></td>
<td></td>
<td></td>
<td>56407.0</td>
<td>71864.0</td>
<td>1276.3</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>280 70 00 30 900 00 70 70 380 00 30 90 00 20 19 20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>47650.0</td>
<td>64999</td>
<td>9557</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>9 3</td>
<td>512 523 411 36 634 594 709 34 695 653 456 35</td>
<td></td>
<td></td>
<td></td>
<td>63703.0</td>
<td>46721.0</td>
<td>354.86</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>61335.0</td>
<td>67314</td>
<td>492</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>10 4</td>
<td>958 531 712 31 834 573 853 33 768 776 864 35 792 813 702 34</td>
<td></td>
<td></td>
<td></td>
<td>63945.0</td>
<td>79182.0</td>
<td>334.47</td>
<td>0.0 0.0 0.0</td>
</tr>
<tr>
<td>0 0 20 25 7 00 40 40 5 00 00 60 0 0 0 0 50 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>42062.0</td>
<td>78477</td>
<td>552</td>
<td>0.0 0.0 0.0</td>
</tr>
</tbody>
</table>
3.4. Optimum processing plant location of Aljalamid phosphate ore.
The mathematical model was used to calculate the optimum location of the processing plant. The required data to apply the mathematical model are the reserves and coordinates of the gravity centers of the different Aljalamid phosphate ore deposits. These data are given in Table 1. According to the output of the FORTRAN program as shown in Fig. 8, the coordinates of the optimum location of the processing plant are approximately (278842, 223149, 735) in the east, north, and elevation directions, respectively. Fig. 9 shows that the calculated optimum location of the processing plant related to the different ore deposit locations of Aljalamid phosphate ore.

![Fig. 8. A snapshot of the end step and required optimum location of processing plant](image)

![Fig. 9. Calculated optimum location of the processing plant related to the different ore deposit locations of Aljalamid phosphate ore.](image)

3.3. Effect of processing plant location deviation from optimum
The hitherto presented results showed the optimum mineral processing plant location in an ideally theoretical case. Due to a reason or another, it may be impossible to install the mineral processing plant at the determined optimum location. Now, it is of importance to investigate the transportation of ore to any location somehow around the optimum location of processing plant. Off course this deviation from the optimum plant location will be reflected on the overall transportation cost. Total
transportation cost of the ore to processing plant location can be calculated by the following equation.

\[ C = c \sum_{i=1}^{n} Q_i \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} \] ..........................(1.5)

Where:
- \( C \): is total transportation cost of the ore for the different locations to the processing plant location in US$.
- \( c \): is transportation cost of one ton for one kilometer distance, in (US$ / ton.Km).
- \( n \): The number of ore deposit locations.
- \( Q_i \): The reserves of each ore deposit location.
- \( x, y \) & \( z \): The coordinates of the processing plant location.
- \( x_i, y_i \) & \( z_i \): The coordinates of center of gravity of each ore deposit locations.

The percent additional cost can be calculated as the cost difference related to the cost to the optimum location and it can be mathematically expressed as follows:

\[ \text{Additional Cost Percent} = \frac{C - C_{opt}}{C_{opt}} \times 100 \] ..........................(16)

Where:
- \( C \): is total transportation cost of the ore for the different locations to the processing plant location in US$. (see Eq. 15).
- \( C_{opt} \): is the total ore transportation cost to the optimum processing plant location, It can be calculated from Eq. 15, when \( x, y \) and \( z \) coordinates refer to optimum processing plant location i.e \( C_{opt} \) is a special case of \( C \) when transportation of ore is going to be to the optimum mineral processing location.

Fig. 10 shows a contour map of the percentage additional cost compared to the minimum for different selected mineral processing plant locations. It shows that there may be a 50% increase in the ore transportation cost due to an incorrect selection of the processing plant location.
IV. CONCLUSIONS

From this study, the following conclusions can be drawn:

1. The phosphate ore in Aljalamid area is a scattered deposit with three main locations that are: fish area, southern area and western area having ore deposits of 451, 362, and 339 million tons of phosphate ore respectively.

2. The gravity centers of the different ore deposit locations were determined and found to be of approximately 55 km far from each other.

3. The suggested mathematical model satisfies a minimum cost of ore transportation to a certain location from different ore deposit locations.

4. Fortran computer program was developed to solve the proposed mathematical model and it was valid.

5. Run of Fortran program shows that coordinates of optimum location of processing plant are (278842, 223149, 735) in the east, north, and elevation directions, respectively.

6. Deviation of processing plant location from optimum may be a 50% increase in the ore transportation cost.

7. The suggested mathematical model can be applied in similar ore deposits.
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