REVIEW ON CONSTRUCTION OF PARITY CHECK MATRIX FOR LDPC CODE

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Abstract— LDPC codes have become the most popular error control code in various fields like telecommunication, magnetic recording, optical communication etc. due to their high error correcting capability. This paper outlines a work on a design of parity check matrix for regular LDPC codes. In this paper we represents an algebraic method to construct parity check matrix for decoding of LDPC code by using weight of row and weight column of matrix. In this paper we have done encoding of LDPC code and representation of parity check matrix by using tanner graph [8]. The construction of LDPC code determines how good the decoding performance and hardware implementation will be. This algebraic method is used in decoding of LDPC code which will be use in sum product algorithm operating on Tanner Graph of LDPC code. The structure of Tanner graph improves the performance in the decoding of LDPC code.

Keywords— Low density parity check (LDPC) code, Tanner Graph, Parity check matrix.

I. INTRODUCTION

Low density parity check codes are a special type of error correcting codes that is known for their good decoding performance and high throughput. Low density parity check codes were first introduced by Robert Gallager in early 60's [1]. His work was ignored for decades because of its high computational complexity for hardware implementation in that time.

LDPC codes are decoded using a subclass of message passing algorithms introduced in Gallager's work named the belief propagation decoding algorithm. Its strength is in the inherent parallelism of the message passing and the iterative decoding [3] behavior that is done by updating bit probabilities. The TG of a linear block code is a bipartite graph constructed from the parity-check matrix of the code. LDPC codes are designed starting from the parity check matrix, where two sets of separated nodes called check and variable nodes are connected to points in the other set based on some regulations and restrictions. The separation of sets allows parallel decoding computations. In contrary, the decoding operations of turbo codes which are the most competitors to LDPC codes, depends on each other in blocks or windows which results in serial computations. LDPC codes have simple graphical representation based on Tanner graph [8] that leads to accurate analysis of performance, also it helps optimizing the designs of regular and irregular constructions. LDPC codes are said to be regular if \( W_c \) is constant for every column, and \( W_r = W_c \times (n/m) \). If the parity matrix \( H \) is low density but the number of “1” in each row or column are not constant, the code is said to be an irregular one.

II. ENCODING OF LDPC CODE

LDPC codes stem from another type of FEC scheme called linear block codes. To describe how LDPC codes operate, consider how typical linear block codes operate. To describe linear block codes, \((n, k)\) notation is commonly used. The \( k \) value denotes the number of bits in a message and the \( n \) value denotes the number of bits in the transmitted codeword. All linear block codes, including LDPC, utilize an encoder matrix, \( G \), that adds redundant information to a message to be transmitted. The redundant information is used to detect and correct errors at the receiver. This redundant
information is embedded in the extra bits called parity bits that are appended to a message. For \((n, k)\) code, the encoding is the process in which k-bit message is augmented with ‘n-k’ parity-check bits to form an n-bit LDPC codeword [7].

Let ‘m’ be message; Then LDPC code for the message shall be \(C = m \times G\).
Where,
‘G’ is Generator matrix
Generator matrix ‘G’ is
\[
G_{k \times n} = [I_k \ P_{k \times (n-k)}]
\]
Where,
\(G\) is a generator matrix of size, \(k \times n\)
\(I\) is an identity matrix of size, \(k \times k\)
\(P\) is a parity bit matrix of size, \(k \times (n-k)\)
The ‘G’ matrix is derived from Parity Check matrix where,
\[
H_{(n-k) \times n} = [P^T \ I_{n-k}]
\]
As matrix G and H are orthogonal to each other,
\(H \times G^T = 0\).
Consider an example: \(m = [1 \ 1 \ 0]\) and \(G\) is given as below
\[
G = \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 1
\end{bmatrix}
\]
Solution: \(C = m \times G\)
\[
= [1 \ 1 \ 0] \times \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 1
\end{bmatrix}
\]
\[
C = [1 \ 1 \ 0 \ 0 \ 1 \ 0]
\]
Thus the codeword contains first three bit message & last three parity bits.

**III. REPRESENTATION OF H MATRIX BY USING TANNER GRAPH**
The Tanner graph of the parity check matrix \(H\) is a bipartite graph. It has bit nodes or variable nodes (VN) equal to the number of columns of \(H\), and check nodes (CNs) equal to the number of rows of \(H\). If \(H_{ji} \neq 1\); i.e., if variable i participates in the jth parity check constraint, then check node j is connected to variable node i.

![Fig.1. Representation of H matrix by using tanner graph](image_url)
Consider $H$ matrix as

$$H = \begin{bmatrix}
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 
\end{bmatrix}$$

**IV. ALGEBRAIC METHOD TO CONSTRUCT PARITY CHECK MATRIX**

Consider an identity matrix $I_a$ where $a = (w_c - 1)(w_r - 1)$ and obtain the matrix by cyclically shifting the rows of the identity matrix $I_a$ by one position to the right.

$$A = \begin{bmatrix}
0 & 1 & 0 & 0 & \cdots & 0 \\
0 & 0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 0 & 1 & \cdots & 0 \\
1 & 0 & 0 & 0 & \cdots & 0 
\end{bmatrix}$$

Defining $A^0 = I_a$ the parity check matrix $H$ can be constructed as

$$H = \begin{bmatrix}
A^0 & A^0 & A^0 & \cdots & A^0 \\
A^0 & A^1 & A^2 & \cdots & A^{(w_c-1)} \\
A^0 & A^{(w_c-1)} & A^{2(w_c-1)} & \cdots & A^{(w_c-1)(w_r-1)} \\
A^0 & A^{(w_r-1)} & A^{2(w_r-1)} & \cdots & A^{(w_c-1)(w_r-1)}
\end{bmatrix}$$

**Example:** Construct $H$ matrix with $w_c = 2$ and $w_r = 3$ using algebraic construction method.

**Solution:**

$(w_c - 1)(w_r - 1) = 2$

Since $a > 2$ then $a = 3$. Hence $I_a = I_3$.

$$A^0 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 
\end{bmatrix}$$

$$A^1 = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 
\end{bmatrix}$$

$$A^2 = \begin{bmatrix}
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0 
\end{bmatrix}$$
\[ H = \begin{bmatrix} A^0 & A^0 & A^0 \\ A^0 & A^1 & A^2 \end{bmatrix} \]

\[
H = \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

V. RESULTS
Fig. 3 shows a simulation result of LDPC encoder. Encoding of LDPC code is done. After the message is encoded, it is transmitted through a channel where it might be corrupted by noise. At the decoder, a corrupted codeword (\( r \)) is received and will be decode by using the \( H^T \) matrix. At the decoder, a corrupted codeword (\( r \)) is received and decoded using the \( H^T \) matrix. This operation results in a value called the syndrome (\( S \)).

If the syndrome is equal to zero, there are no errors detected. For this result we are working on Sum product algorithm for decoding of LDPC.

VI. CONCLUSION
Construction gives a large class of regular LDPC codes in Gallager’s original form that perform well with the SPA. The best known practical algorithm for the decoding of low-density parity-check (LDPC) codes is the iterative sum-product or belief propagation algorithm, operating on a Tanner graph (TG) of the code. In this paper, we proposed algebraic construction of parity check matrix \( H \) of regular LDPC code. From this \( H \) matrix we will decode the received codeword by using Sum product algorithm [5]. It improve the performance or complexity trade-off in the decoding of short LDPC codes.

REFERENCES

