Stress Distribution Analysis in Non-Involute Region of Spur Gear

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Abstract — This research paper highlights the critical sections and demonstrates the undercutting phenomenon that might happen in production of gear from gear blanks, or during run time of the pair and can lead to failure of the gear. The research paper highlights the distribution of stress in non-involute region between the base circle and Dedendum circle, which happens to be a reason of interference during the gear pair in motion. Also the authors have tried to investigate the extent of deformation and magnitude of strains produced for a given problem using CAD modeling and FEA.

I. INTRODUCTION
Involute tooth profiles are the best for gear mechanisms since they provide constant velocity ratio, as there is no slip between gears. Although cycloid tooth profiles also give positive drive to gear pair, but they are less popular as their cost is high and manufacturing is complex because of two curves involved. Further a fixed center distance between shafts is must in cycloid gears. Whereas in involute gears any variation in center distance (due to shaft deflection and manufacturing tolerances) will not affect uniform motion. In involute gears, the pressure angle from the start of engagement of teeth to the end of engagement remains constant. Against this in cycloid gears, pressure angle varies from the beginning to the end of engagement. Involute curve is the path traced by a point on a line as the line rolls without slipping on the circumference of a circle (Base Circle). Thus it is clear that involute profile is from base circle to gear top. But the region between base circle and dedendum circle is non-involute. And any contact (if happens in this region) will lead stress generation. The authors have tried to investigate these stresses (their magnitude and intensity), also the tentative deformation and its extent. For the sake of studies’ a theoretical problem is selected, based on which a CAD model is prepared on CATIA v-5(full version) and then analysis ids carried using Ansys 17.2 Workbench, student’s version.

II. DESIGN OF SPUR GEAR
2.1 Design Problem
Referencing one question that can be accepted as a practical application of gears, taken from Machine Design book by V. B. Bhandari is as follows,

It is required to design a 20° full depth involute teeth based on Lewis Equation. The velocity factor is to be used for dynamic loading. The pinion shaft is connected by 10 KW, 1440 rpm motor. The starting torque of the motor is 150% of the rated torque. The speed reduction is 4:1. The pinion as well as gear is made of plain carbon steel 40C8 (Sₜₚₚ = 600 N/mm²). The factor of safety can be taken as 1.5. Design the gears specify their dimensions.

The analysis can be carried on different categories of gears, having different dimensions and operating under different conditions. For the sake of standardization, the authors have selected a theoretical problem from a standard text book ‘Design of Machine Elements’ by V.B. Bhandari. A CAD model of the problem is prepared and F.E.A. analysis is performed on CATIA and ANSYS respectively.
2.2 Design Equations and theory
The following equations are used for designing a spur gear tooth,
1. Lewis Equation and AGMA Strength Equation: It models a gear tooth taking the full load close to its tip as a simple cantilever beam.

\[ \sigma = \frac{F_t P}{bJ} K_v K_d K_m \]

\( F_t \) = Transmitted force,
\( P \) = Applied load,
\( K_v \) = Velocity or dynamic factor,
\( K_d \) = Load factor,
\( K_m \) = Mounting factor,
\( b \) = Face width.

2. Pitch circle diameter: \( R_p = \frac{(m \times N)}{2} \)
3. Base circle diameter: \( R_b = 0.94 \times R_p \)
4. Addendum: \( R_a = R_p + m \)
5. Dedendum: \( R_d = R_p - (1.25 \times m) \)

2.3 Results:
1. Number of teeth on pinion: 18.0
2. Module: 5.0 mm
3. Pitch circle diameter: 90.0 mm
4. Base circle radius, \( R_b \) 42.3 mm
5. Addendum, \( R_a \) 50.0 mm
6. Dedendum, \( R_d \) 38.75 mm
7. Acting force: 7206 N

2.4 3-D model results on Catia
The above given results are then modelled on the software and the following graphic results are obtained:

Profile made using the parametric equations:

![Fig. 2: Gear Tooth Profile](image1)

3-D model on Catia-V5:

![Fig. 3: Gear model on Catia V-5](image2)
Part of model required for analysis:

![Fig. 4: required part of the model.](image)

2.5 Analysis carried on Ansys

The 3-D model constructed on Catia is transferred to Ansys in an ‘.igs’ format file. The material given in main question is 40C8, but not restricting to the only one material, but having multiple choices that are practically used in gear production we opt EN-JL-1020, grey cast iron, as our trail material and use 40C8 later for simulation under same cases. Hence we get the following results (iteration carried to achieve less than 1% convergence)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile Strength</td>
<td>Rm</td>
</tr>
<tr>
<td>0.1% Yield Strength</td>
<td>Rp0.1</td>
</tr>
<tr>
<td>Compressive Strength</td>
<td>σdB</td>
</tr>
<tr>
<td>0.1% Compressive Strength</td>
<td>σd0.1</td>
</tr>
<tr>
<td>Modules of elasticity</td>
<td>E</td>
</tr>
<tr>
<td>Poisson number</td>
<td>v</td>
</tr>
<tr>
<td>Density</td>
<td>ρ</td>
</tr>
<tr>
<td></td>
<td>MPa</td>
</tr>
<tr>
<td></td>
<td>MPa</td>
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<td>MPa</td>
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<tr>
<td></td>
<td>MPa</td>
</tr>
<tr>
<td></td>
<td>GPa</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>g/cm³</td>
</tr>
</tbody>
</table>

*Table 1: Details of EN-JL-1020*

![Fig. 5: Model Transfer in Ansys, ready in modelling frame.](image)

![Fig. 6: Adding material to the Engineering materials library](image)

![Fig. 7: Normal results without any additive mesh controls.](image)
Results of iteration:

<table>
<thead>
<tr>
<th>Sol. No.</th>
<th>Action</th>
<th>Nodes</th>
<th>Maximum Principle Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Normal force application</td>
<td>4358</td>
<td>23.487</td>
</tr>
<tr>
<td>2</td>
<td>Face Meshing</td>
<td>4358</td>
<td>23.487</td>
</tr>
<tr>
<td>3</td>
<td>Refinement</td>
<td>8815</td>
<td>44.19</td>
</tr>
<tr>
<td>4</td>
<td>Refinement= 2</td>
<td>14313</td>
<td>51.803</td>
</tr>
<tr>
<td>5</td>
<td>Relevance Centre, Coarse to Medium</td>
<td>28918</td>
<td>69.596</td>
</tr>
<tr>
<td>6</td>
<td>Face Sizing = 3.5</td>
<td>29072</td>
<td>65.608</td>
</tr>
<tr>
<td>7</td>
<td>Relevance Centre= 2</td>
<td>28866</td>
<td>68.822</td>
</tr>
<tr>
<td>8</td>
<td>Face Sizing = 3.2</td>
<td>29411</td>
<td>69.208</td>
</tr>
<tr>
<td>9</td>
<td>Edge Sizing= 2.7</td>
<td>31940</td>
<td>69.148</td>
</tr>
</tbody>
</table>

Table 2: Results of iteration

Last two solutions have a convergence of 0.08% hence the obtained results can be accepted and used for simulation of the material as given in the main problem.

Graph 1: Stress Strain Graph for EN-JL-1020

As given in the main problem, we now change the material applied on the test model to 40C8, of which the details are:
Fig. 10: Applying material 40C8 to the test model.

The results of the iteration:

<table>
<thead>
<tr>
<th>Sol. No.</th>
<th>Action</th>
<th>Nodes</th>
<th>Maximum Principle Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Normal force application</td>
<td>4358</td>
<td>23.849</td>
</tr>
<tr>
<td>2</td>
<td>Relevance Centre 2</td>
<td>4358</td>
<td>23.849</td>
</tr>
<tr>
<td>3</td>
<td>Face Sizing 3 mm</td>
<td>5356</td>
<td>23.895</td>
</tr>
<tr>
<td>4</td>
<td>Face Sizing 2.5 mm</td>
<td>5861</td>
<td>38.522</td>
</tr>
<tr>
<td>5</td>
<td>Refinement 1</td>
<td>10030</td>
<td>68.305</td>
</tr>
<tr>
<td>6</td>
<td>Refinement 2</td>
<td>20187</td>
<td>85.381</td>
</tr>
<tr>
<td>7</td>
<td>Face Meshing</td>
<td>19119</td>
<td>84.455</td>
</tr>
<tr>
<td>8</td>
<td>Meshing Relevance Centre Medium</td>
<td>25985</td>
<td>80.764</td>
</tr>
<tr>
<td>9</td>
<td>Face Sizing Broader Area 5mm</td>
<td>25908</td>
<td>82.105</td>
</tr>
<tr>
<td>10</td>
<td>Edge Sizing 2.5mm</td>
<td>25908</td>
<td>82.105</td>
</tr>
</tbody>
</table>

Table 4: Results of iteration for 40C8

Fig. 11: Force application over the non-involute area.

Table 3: Details of 40-C-8

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile Strength $R_m$</td>
<td>660</td>
</tr>
<tr>
<td>0.1% Yield Strength $R_{p0.1}$</td>
<td>560</td>
</tr>
<tr>
<td>Modules of elasticity $E$</td>
<td>210</td>
</tr>
<tr>
<td>Poisson number $v$</td>
<td>0.30</td>
</tr>
<tr>
<td>Density $\rho$</td>
<td>7.85</td>
</tr>
</tbody>
</table>

Fig. 12: The results of simulation
The comparison in the curves make this a clear understanding that under same working conditions of undercutting, every material behaves in its own particular manner but essence remains the same which can be understood as, ‘in the region of undercutting the stress along with strain increases linearly’. The graphs also considers proof stress which is the reason for graph being present in negative region.

The reason for linear increment can be understood as a phenomenon where adjacent tooth touches the teeth at the non-involute profile sectors and undercutting comes into picture parallel to interference. The strain in the tooth begins as contact begins and the applied pressure increases as there remains no free space for the tooth to revolve out of contact.

Differing to density and Young’s modulus the slope of Stress Strain Curve differs.

### III. CONCLUSION

As the purpose of the research work is to study the distribution of stresses in non-involute region, results derived are expressed as pictures (visual outcomes) shown above that displays the severities of Maximum Principle Stress in different regions prone to failure.

Hence, for different materials the different results are obtained in the terms of Maximum Principle Stress and Strain, the magnitude of these values in the most critical zone is as follows:

<table>
<thead>
<tr>
<th>S. No</th>
<th>Material Used</th>
<th>Maximum stress (MPa)</th>
<th>Corresponding Strain (mm/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>EN-JL-1020</td>
<td>69.148</td>
<td>8.41E-03</td>
</tr>
<tr>
<td>2</td>
<td>40C8</td>
<td>82.105</td>
<td>3.84E-03</td>
</tr>
</tbody>
</table>

*Table 5: Results obtained after simulation*
IV. THEORITICAL UNDERSTANDING OF ELIMINATING INTERFERENCE

Interference and hence undercutting in gears can be avoided by approaching any following methods:
1. Using a higher magnitude pressure angle.
2. Designing a tooth with higher load carrying capacity.
3. Increasing the length of contact.
4. Altering the center to center distance between the pitch circles of the gears in contact.
5. Stubbing the tooth eliminates interference.
6. Undercutting the tooth over non-involute region so that the contact in mating tooth does not happen. Since this method reduces the strength of the tooth, hence in applications of higher power transmission, this method is not used.

V. ACKNOWLEDGMENT

In this paper we would like to present our gratitude towards our guiding spirit, Dr. Ashok G. Ambekar who has taken his great efforts in conceptualizing the basic facts. Dr. Suyog Jhawar who has given his time and efforts in teaching the basic CAD software and the inspiration in going after researches and of publishing every thought process in a research paper.

REFERENCES