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A STUDY ON MINIMUM COVERING ENERGY OF A GRAPH

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Abstract: In this chapter we discuss some basic properties of minimum covering energy and derive an upper bound and a lower bound for $E_c(G)$. Compute the minimum covering energy of path and cycle.

Keywords: $E_{(c)}(G)$ – Minimum covering Energy, $A_{(c)}(G)$ – Minimum covering Matrix

$\sum a_{ii}^2 = |C|$, $\sum a_{ij}^2 = |E|$, $\sum (a_{ii} \quad a_{jj}) = \binom{|C|}{2}$, $\sum (a_{ii} \quad a_{jj} \quad a_{kk}) = \binom{|C|}{3}$.

I. INTRODUCTION

The minimum covering energy of a graph introduced by Chandrashekar Adiga [1,5], is the motivation behind this project. In this dissertation this particular energy is studied. P_n and C_n [8,10,12] for different values of n are considered and their minimum covering energies have been calculated and tabulated. For some theorem have been checked.

II. DEFINITIONS

2.1 Graph

A Graph $G = (V, E)$ consists of a set of objects $V = \{v_1, v_2, v_3 \ldots\}$called vertices and another set $E = \{e_1, e_2, e_3, \ldots\}$ whose elements are called edges, such that each edge $e_k$are called the end vertices of e_k.

The most common representation of a graph is by means of a diagram, in which the vertices are represented as points and each edge as a line segment joining its end vertices.

2.2 Path graph

An open walk in which no vertex appears more than once is called a path.

2.3 Cycle graph

A circuit that does not contain any repetition of vertices except the starting vertex and the terminal vertex is called cycle.

2.4 Energy of a graph

The energy, E (G), of a graph G is defined as the sum of the absolute values of its eigen values.

2.5 Minimum Covering set

A subset C of V is called a *covering set* of G if every edge of G is incident to at least one vertex of C. Any covering set with minimum cardinality is called a minimum covering set.

2.6 Minimum Covering Matrix

Let C be a minimum covering set of a graph G. The minimum covering matrix of G is the $n \times n$matrix$A_c(G) = (a_{ij})$, where

$$a_{ij} = \begin{cases} 1, & if\, v_i v_j \in E \\ 1, & if\ i=j\ and\ v_i \in C \\ 0, & otherwise \end{cases}$$

The characteristic polynomial of $A_c(G)$ is denoted by
$f_n(G,\lambda) = \det(\lambda I - A_c(G))$.

2.7 Minimum Covering Eigenvalues

The Minimum Covering Eigen values of the graph G are the eigenvalues of $A_c(G)$.Since $A_c(G)$ is real and symmetric, its eigenvalues are real numbers and we lable them in non-increasing order $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$.

2.8 Minimum Covering Energy

The Minimum Covering energy of G is then defined as

$$E_c(G) = \sum_{i=1}^{n} |\lambda_i|.$$

III. Compute the minimum covering energy of two graphs

3.1 Path Graph

Graph	Energy	Minimum covering set	Minimum covering Energy
P_4	4.4720	$C_1(P_4) = \{v_1, v_3\}$ $C_2(P_4) = \{v_2, v_3\}$	4.9770 4.8284
P_5	5.4610	$C_1(P_5) = \{v_2, v_4\}$	4.9770
P_6	6.9890	$C_1(P_6) = \{v_2, v_{4,} v_5\}$ $C_2(P_6) = \{v_1, v_3, v_5\}$	7.5728 7.7658
P_7	8.0546	$C_1(P_7) = \{v_2, v_{4,} v_6\}$	8.6568
P_8	9.5175	$C_1(P_8) = \{v_2, v_{3,} v_{5,} v_7\}$ $C_1(P_8) = \{v_2, v_4, v_{6,} v_8\}$	10.3576 10.5664
P_9	10.6275	$C_1(P_9) = \{v_2, v_4, v_{6,} v_8\}$	11.4654
P_{10}	12.0532	$C_1(P_{10}) = \{v_2, v_{4,} v_{4,} v_{4,} v_4\}$ $C_2(P_{10}) = \{v_1, v_{3,} v_{5,} v_{7,} v_9\}$	13.3704 13.3704

3.2 Cycle Graph

Graph	Energy	Minimum covering set	Minimum covering Energy
C_4	6.0000	$C_1(C_4) = \{v_1, v_3\}$ $C_2(C_4) = \{v_2, v_4\}$	5.1232 5.1232
C_5	6.4720	$C_1(C_6) = \{v_1, v_{3,} v_5\}$ $C_2(C_6) = \{v_1, v_2, v_4\}$ $C_3(C_6) = \{v_1, v_{3,} v_4\}$	7.4286 7.4286 7.4286
C_6	8.0000	$C_1(C_6) = \{v_1, v_3, v_5\}$ $C_2(C_6) = \{v_2, v_4, v_6\}$	8.5952 8.5952
C_7	8.9878	$C_1(P_8) = \{v_1, v_{2,} v_{4,} v_6\}$ $C_1(P_8) = \{v_1, v_{3,} v_{5,} v_6\}$	9.7961 9.7961
C_8	9.5176	$C_1(P_8) = \{v_1, v_{3,} v_{5,} v_7\}$ $C_1(P_8) = \{v_2, v_4, v_{6,} v_8\}$	11.1232 11.1232
C_9	10.0276	$C_1(P_9) = \{v_2, v_4, v_{6,} v_8\}$ $C_1(P_8) = \{v_2, v_{3,} v_{5,} v_7\}$	12.4586 12.4586
C_{10}	11.5276	$C_1(P_{10}) = \{v_2, v_{4,} v_{4,} v_{4,} v_4\}$ $C_2(P_{10}) = \{v_1, v_{3,} v_{5,} v_{7,} v_9\}$	13.5689 13.5689

IV. Some basic properties of minimum covering energy

4.1 Theorem

If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of $A_c(G)$, then $\sum_{i=1}^{n} \lambda_i^2 = 2|E| + |C|$.

Proof

The sum of squares of the eigenvalues of $A_c(G)$ is just the trace of $A_c(G)^2$.
Therefore,

$$\sum_{i=1}^{n} \lambda_i^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} a_{ji}$$

$$= 2\sum_{i<j} (a_{ij})^2 + \sum_{i=1}^{n} (a_{ij})^2$$

$$= 2|E| + |C|.$$

$$\sum_{i=1}^{n} \lambda_i^2 = 2|E| + |C|. \quad (1)$$

4.2 Theorem

Let G be a graph with n vertices, m edges, and let C be a minimum covering set of G. Then

$$E_c(G) \le \sqrt{n(2m + |C|}.$$

Proof

Let $\lambda_1 \ge \lambda_2 \ge \lambda_3 \ge \ldots \ge \lambda_n$ be the eigenvalues of Ac (G). Then by Cauchy-Schwarz inequality, $(\sum_{i=1}^{n} a_i b_i)^2 \le (\sum_{i=1}^{n} a_i^2)(\sum_{i=1}^{n} b_i^2)$

We choose $a_i = 1$ and $b_i = |\lambda_i|$ we get [by 1]

$$E_c(G)^2 = \left(\sum_{i=1}^{n} |\lambda_i|\right)^2 \le n \left(\sum_{i=1}^{n} |\lambda_i|^2\right)$$

$$= n \sum_{i=1}^{n} \lambda_i^2 = n(2m + |C|).$$

$$E_c(G)^2 = n(2m + |C|). \quad (2).$$

4.3 Theorem

Parity Theorem

Let G be a graph with a minimum covering energy E_c (G) of G is a rational number, then

$E_c(G) \equiv |C| \pmod 2$

Proof

Let $\lambda_1, \lambda_2, \ldots, \lambda_r$ be positive, and the rest of the minimum covering eigenvalues non-positive . Thus

$$E_c(G) = \sum_{i=1}^{n} |\lambda_i| = (\lambda_1+\lambda_2+\ldots+\lambda_r) - (\lambda_{r+1} + \ldots + \lambda_n)$$

Implying

$E_c(G) = 2(\lambda_1+\lambda_2+\ldots+\lambda_r) - |C|$.

Since $\lambda_1, \lambda_2, \ldots, \lambda_r$ are algebraic integers , so is their sum. Hence $(\lambda_1+\lambda_2+\ldots+\lambda_r)$ must be an integer if $E_c(G)$ is rational.

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