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Efficient Time-Frequency Domain Algorithm for Real Time Signal Compression

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Abstract: For real time remote health monitoring systems, spontaneous transmission of biological signals is essential. To accomplish this time efficient compression schemes play an important role. In this paper, we propose a sparse encoding algorithm consisting of a wavelet transform based iterative thresholding (WTIT). It reduces the minimal ECG voltage values to zero level. Subsequently, it encodes the ECG signal in time-frequency domain, obtaining a high sparsity level. Compressed Row Huffman Coding (CRHC) algorithm converts the sparse matrices into compressed, transmittable matrices. We apply inverse transforms to reconstruct the transmitted signal and test the performance of encoding and reconstruction in terms of compression ratio (CR), percentage root mean square difference (PRD) and time complexity.

Keywords: Remote health monitoring systems, Real time, Wavelet Transform, Iterative thresholding, Transmittable matrix

I. INTRODUCTION

A typical ECG monitoring device generates volumes of digital data creating a necessity for efficient compression before real time transmission of the signal. Most of the prior research [1-3] assumed ECG signal to be noiseless and would possess definite sparsity levels. However, during ECG signal acquisition, the sparsity level may vary due to mass motion. Also, additive Gaussian noise remains in signal which results into powerline interferences and baseline drifts [4].

For data compression, we have used parameter extraction and wavelet based transformation methods. The sparsity of ECG signal, that can be achieved, is examined to realize the compressibility of the signal through our proposed algorithm involving sparsity accomplishment scheme: WTIT followed by lossless encoding scheme *CRHC*.

While most of the related earlier works [2,3,5] either achieved low PRD, or high CR; in this work, we have achieved lower CR and PRD simultaneously. Also, previous work [6] on ECG compression using wavelet transform assumed an arbitrary threshold value, but the threshold may change in ECG signal due to various cardiovascular diseases. In this paper, the algorithm WTIT determines local threshold depending upon the noise variance at a region.

In time-frequency domain, the threshold changes from time to time across the ECG transformed matrices and thus, it is effective in case of any cardiovascular disease a patient might be affected with.

In this work, our contribution is to develop an end-to-end model, which (i) filters out noise from the raw ECG signal, (ii) encodes the noiseless signal in form of highly sparse matrices, (iii) converts the sparse matrices into transmittable matrices of much reduced size, (iv) reconstructs the signal using inverse transformations.

For real time signal transmission, the processing time plays the key role. The time complexity of our proposed algorithms is in the order of $O(n)$, while most the previous related works had time complexities in the order of $O(n^2)$.

Section II describes our proposed method while section III discussed results. Section IV concludes.

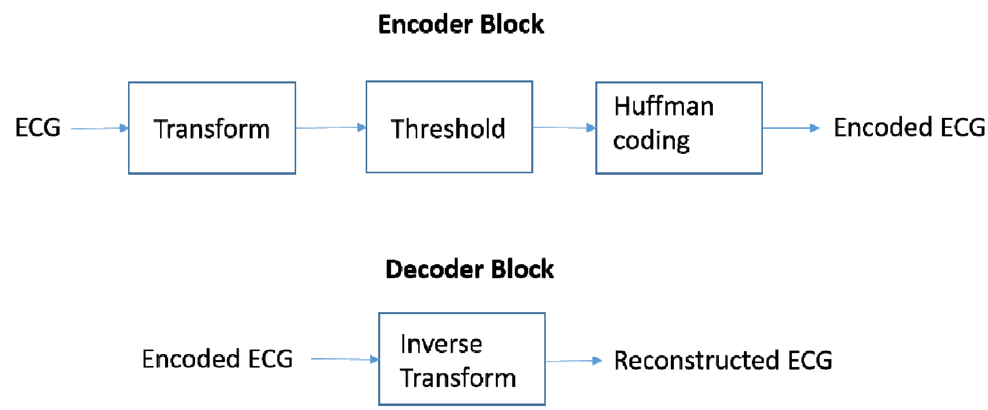

Figure 1. General transform based compression

II. FRAMEWORK FOR THE COMPRESSION OF ECG SIGNAL

Fig. 1 shows the framework on which a set of algorithms are applied for the compression of ECG signal in real time.

The continuous wavelet transform $\boldsymbol{X}(a,b)$ of any signal $\boldsymbol{x}(t)$ at time t can be represented as:

$$\boldsymbol{X}(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} \boldsymbol{x}(t)\boldsymbol{\Psi}^{*}\frac{(t-b)}{a}dt \ , \qquad -\infty < t < \infty \qquad (1)$$

Here, a is the scaling factor, b is the translation factor, $\boldsymbol{\Psi}^{*}(t)$ is the complex conjugate of the mother wavelet $\boldsymbol{\Psi}(t)$.

In discrete wavelet transform, the signal $\boldsymbol{x}[n]$, which is a discrete time signal such that $\boldsymbol{x}[n] = \boldsymbol{x}(nT)$ where $\frac{1}{T}$ is the sampling frequency, is continuously passed through a set of high pass and low pass filters, $\boldsymbol{h}[n]$ and $\boldsymbol{g}[n]$ respectively as shown in Fig. 2.

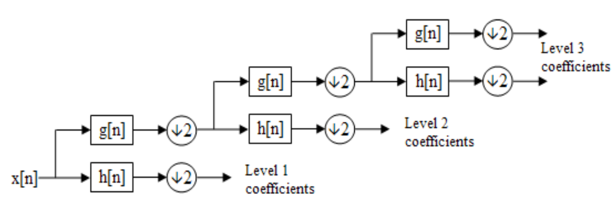

Figure 2. DWT of x[n] upto three levels of decomposition

The output of the high pass filter after down sampling by a factor of two yields detailed coefficients ($\boldsymbol{D}$) and the output of the low pass filter after down sampling by a factor of two yields approximate coefficients ($\boldsymbol{A}$).

$$\boldsymbol{D_1} = \sum_{k=-\infty}^{\infty} \boldsymbol{h}[k]\boldsymbol{x}[2n-k] \qquad (2)$$
$$\boldsymbol{A_1} = \sum_{k=-\infty}^{\infty} \boldsymbol{g}[k]\boldsymbol{x}[2n-k] \qquad (3)$$

$$\boldsymbol{D_j} = \sum_{k=-\infty}^{\infty} \boldsymbol{h}[k]\boldsymbol{A_{j-1}}[2n-k] \quad (4)$$
$$\boldsymbol{A_j} = \sum_{k=-\infty}^{\infty} \boldsymbol{g}[k]\boldsymbol{A_{j-1}}[2n-k] \quad (5)$$

where $0 \leq n \leq l$, $\boldsymbol{D_j}$ and $\boldsymbol{A_j}$ represents j^{th} level detailed and approximate coefficients respectively, $0 \leq j \leq log_2 l$. l is the length of the analyzing signal.

Denoising:
Our purpose is to develop a noise tolerant algorithm to evaluate the sparsity and make ECG signal transmission ready.

Maximum-Likelihood adaptive filters have been for partially-observed Boolean dynamic systems which uses combination of Boolean kalman filter and Booleankalman smoother [13]. The maximum likelihood estimation have been performed using expectation-minimization algorithm.

Wavelet based denoising is done by decomposing the signal using band pass filters corresponding to a mother wavelet basis function. This stage corresponds to a preprocessing with the raw ECG signal as the input. We set decomposition level dependent threshold value(T_l) as per Donoho's universal thresholding [7]:

$$T_l = \sigma\sqrt{2\log(l)} \quad (6)$$

where σ is noise variance of the signal at each level of decomposing.

Let for a given signal $\boldsymbol{u}[n]$, there exists an orthogonal matrix $\boldsymbol{W}$ such that the discrete wavelet transformed matrix $\boldsymbol{U}[n]$ is given by:

$$\boldsymbol{U} = \boldsymbol{W}\boldsymbol{u} \quad (7)$$

Let the additive noise to signal$\boldsymbol{x}[n]$ be $\boldsymbol{y}[n]$ which has a noise variance σ^2. Hence, the resultant signal $\boldsymbol{g}[n]$ is now:

$$\boldsymbol{g}[n] = \boldsymbol{x}[n] + \boldsymbol{y}[n] \quad (8)$$

The inverse wavelet transform can be easily computed as:

$$\boldsymbol{u} = \boldsymbol{W}^T\boldsymbol{U} \quad (9)$$

The wavelet transformed matrix $\boldsymbol{U}[n]$ is now modified to $\widetilde{\boldsymbol{U}}[n]$ by applying threshold and as a result $\boldsymbol{u}[n]$ becomes $\widetilde{\boldsymbol{u}}[n]$

$$\widetilde{\boldsymbol{u}} = \boldsymbol{W}^T\widetilde{\boldsymbol{U}} \quad (10)$$

Reduction of wavelet transformed coefficients produces significantly noise free estimates, which can be done either by hard or soft thresholding.

$$T_{hard}(\widetilde{\boldsymbol{U}}; T_l) = \boldsymbol{U}O(|\boldsymbol{U}| > T_l) \quad (11)$$

$$T_{soft}(\widetilde{\boldsymbol{U}}; T_l) = sgn(\boldsymbol{U})(\boldsymbol{U} - T_l)O(|\boldsymbol{U}| > T_l) \quad (12)$$

O is the usual indicator function and sgn represents signum function. For noise removal, we consider soft thresholding of wavelet coefficients as hard thresholding can result into significant information loss from the original signal.

The noise free ECG signal is free from zero mean band limited Gaussian noise i.e., the minute interferences present in the signal but they are not entirely free from the periodic baseline drifts. The noiseless ECG signal is then processed using our encoding algorithm.

Encoding Algorithm:

A typical single cardiac cycle waveform of a normal heartbeat is shown in Fig 3. The first upward deflection P is the atrial complex with low amplitude level. Following the P wave, comes the QRS complex (ventricular complex), with the largest voltage deflection. The T wave, again has a low amplitude level. We observe that no R-R wave can occur in more than 1.2 second distance on the time axis from physiological point of view after conducting review on a considerable number of ECG signals obtained from the MIT-BIH database [8]. This 1.2 second distance on time scale corresponds to approximately 85-90 samples with respect to the sampled signal obtained from MIT-BIH database. Only the amplitude of R wave is found to be greater than *0.5mV.*

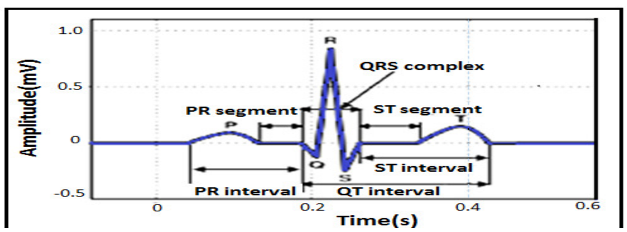

Figure 3. A Typical ECG Waveform

Encoding by wavelet transform:

Various encoding techniques uses samples available from previously simulating disciplines by applying sequential resampling. The absence or lack of samples in each discipline is addressed by introducing an adaptive greedy sample increment process to improve the efficiency of uncertainty analysis with minimum possible computational cost [14].

After denoising, the ECG signal is decomposed by wavelet transformation (eqns. 2-5). We use WTIT to extract the high frequency components of ECG signal. This WTIT automatically sets high threshold value for lower level coefficient and lower threshold to higher level coefficient. This will be continued to reach higher sparsity in ECG signal, keeping the higher frequency components intact. In WTIT, a window function of fixed lengthl', slides over each of the wavelet transformed matrices obtained from the ECG signal. The universal threshold (Y) of the transformed signal within the window is calculated as per eqn.6. Thresholding is performed as per Eqn. 11. Next the window is shifted to another point in the transformed matrix and the process is repeated.

Encoding algorithm:*Wavelet Transform based iterative thresholding (WTIT)*

Input:$\boldsymbol{\alpha}$ is a column vector containing wavelet transform coefficients of ECG signal of a decomposition level, Y calculates universal threshold as per eqn.6.

Initialize:$\boldsymbol{\omega}$=null vector, m=0, n=0, l' = length of window, μ is iterative threshold and I and y are counter variables.

Step I: *Repeat*

Set m to m + 1

Set n to l'-(m -1)*n

Set *I* to 0

Step II: *Repeat*

Set **ω**[*I*] to ***α***[*I*]

Increment *I* by one

Until I is <n

Step III: Set μ to Y of **ω**[*I*]

Set y to (m-1)* n

Step IV: *Repeat*

If □[y]is ≤ μ

Set □ [y] to 0

Increment y by one

Until □≤□ + (m-1)* n

Set $\square'$ to $\square'$ +n

Until $\square'$< length of vector□

Let us consider, threshold of any wavelet decomposed level $\square_{\square}$,∀j. For the sake of simplicity, we consider $\square'$ as an even number. When sliding window is centered at a sample numbered □, the matrix to be thresholded is represented as $\square_{\square,\square}$.

$$\square_{\square,\square_{\square}} = \begin{pmatrix} \square_{\square}[\square_{\square} + \frac{\square'}{2}] \\ \vdots \\ \vdots \\ \square_{\square}[\square_{\square} - \frac{\square'}{2}] \end{pmatrix} \qquad (13)$$

Now,

$$\bar{\square}_{\square,\square_{\square}} = \square_{\square_{\square,\square_{\square}}} \sqrt{2 \log (\square')} \qquad (14)$$

where $\bar{\square}_{\square,\square_{\square}}$ is the universal threshold of $\square_{\square,\square_{\square}}$. If the iterative threshold form of $\square_{\square,\square_{\square}}$ is $\tilde{\square}_{\square,\square_{\square}}$ then,

$$\tilde{\square}_{\square,\square_{\square}}(\square) = \square_{\square,\square_{\square}}(\square) \quad \square\square\square \left|\square_{\square,\square_{\square}}(\square)\right| \geq \bar{\square}_{\square,\square_{\square}}.$$
$$\tilde{\square}_{\square,\square_{\square}}(\square) = 0 \qquad \square\square\square \left|\square_{\square,\square_{\square}}(\square)\right| < \bar{\square}_{\square,\square_{\square}} \qquad (15)$$

Essentially, we obtain *a high amount of sparsity of ECG signal without considerable loss in the high frequency components through our encoding algorithm.*

Conversion of sparse matrix into transmittable matrix using CRHFC:

A requisite for a transmittable signal is the compression of a highly sparse column matrix to a matrix containing fewer non-zero elements to minimize the transmission time. After applying the discrete wavelet transform in the WTIT algorithm, we obtain matrices containing detailed and approximate coefficients. Before transmission (refer to Fig. 1), all these matrices can further be compressed by using generalized method [9]. In our proposed CRHC algorithm, each of the coefficient row matrices are converted into square matrices of order ($\square_1 \square\, \square_1$). To convert a row matrix into a square matrix, ($\square_1^2 - \square$) zeroes are appended to each of the row matrix, when,

Case 1: $□_{1} = \sqrt{□}$, when n∈□, where Z is set of positive integers.
Case 2: $□_{1} = \lceil \sqrt{□} \rceil + 1$, when $□ \in (□ - □)$, where R is set of positive real numbers.

Algorithm: *Compressed Row Huffman Coding (CRHC)*
Input: The row matrix with $□_{1}^{2}$elements: [**E**]; let $□_{1,j}$ be the j[th]
element. The square matrix [**C**] formed from [**E**] in the primary step; let $□_{□,□}$ be its element of $□^{□h}$ row and $□^{□h}$ column.

$$□ = \begin{bmatrix} □_{1,1} & □_{1,2} & □_{1,3} & \cdots\cdots\cdots & □_{1,□_1} \\ □_{2,1} & □_{2,2} & □_{2,3} & \cdots\cdots\cdots & □_{2,□_1} \\ □_{3,1} & □_{3,2} & □_{3,3} & \cdots\cdots\cdots & □_{3,□_1} \\ . & . & . & . & . \\ . & . & . & . & . \\ □_{□_1,1} & □_{□_1,2} & □_{□_1,3} & \cdots\cdots\cdots & □_{□_1,□_1} \end{bmatrix}$$

In this matrix, $\left[□_{□,□}\right] = \left[□_{1,[\{(□-1)\times □_1\}+□]}\right]$ (16)

Step I:*CR:*The compressed row storage algorithm [10] compresses each of the square matrices [**C**] and generates three different row matrices for each individual matrix.

i)The element matrix [**N**]: The non-zero elements of [**C**] as traversed across each row commencing from the top.
ii)The column indicating matrix [**IN**]: The column indices of non-zero elements in [**C**] as traversed across each row commencing from the top.
iii) The row indicating matrix[**JN**]: The location of the non-zero element encountered first in each row commencing from the top.

Step II**:** *HC:*The element row matric [**N**], obtained from the compressed row storage algorithm, is further compressed using Huffman coding [11].
Combination of Step I and Step II gives CRHC.

***Reconstruction*:**
The reconstruction algorithm corresponding to the second last stage i.e., decoding in Fig. 1 consists of a two level algorithm. The procedure begins with decoding of CRHC encoded matrix by means of i) Huffman decoding and performing the converse of compressed row storage algorithm, and terminates by ii) executing the inverse transforms on the coefficient matrices obtained in the first level.

III. PERFORMANCE STUDY AND RESULTS

To validate the performance of the above algorithms enumerated in section II, we use a set of ECG signal samples from MIT-BIH database [8]. The PRD, CR and time complexity are measured through simulation in MATLAB. Fig.5. shows the performance for ECG signal with window of size 1000 samples. The performance of our algorithms is evaluated by calculating average results from 15 ECG signal sets. The encoding algorithm, *achieves a high sparsity level of 87% to 91%.* WTIT does signal processing in time-frequency domain. Hence, WTIT and CRHC are combined, to calculate PRD and CR

Algorithm	PRD	CR
Simultaneous OMP [2]	2.57	7.23
BP [5]	1.66	4
BP [3]	<9	3.44
BP [12]	9	2.5
Our encoding algorithm	2.3	9.1

Table 1.Comparison among ECG compression and algorithms with our algorithm

The absolute time complexity of the whole procedure exclusive of the transmission media time is □(□).

IV. CONCLUSION

In this work, we have presented a model comprised of sub algorithms WTIT and CRHC. There are several distinct features to our scheme namely high sparsity, efficient processing time, ease of data storage and reasonable values of data storage and reasonable values of PRD and CR.

Our analysis and simulation results suggest that our algorithm is promising real time transmission of biomedical signal (in this case ECG) due to its efficient time complexity. The hardware implementation of our algorithm will be explored in our future work.

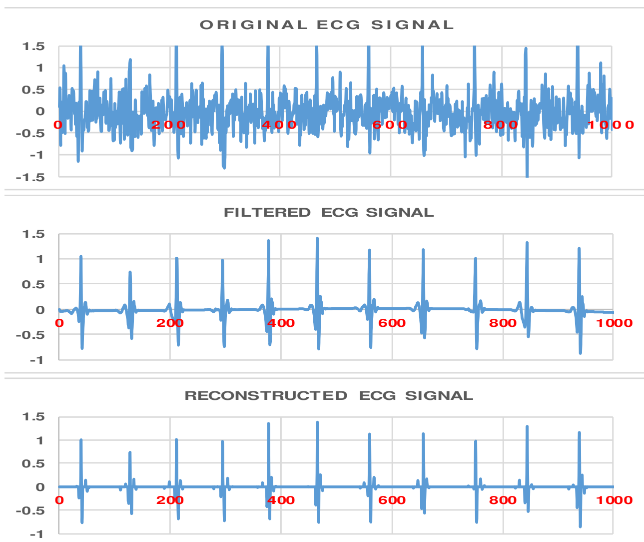

Figure 4.Performances of algorithms over a ECG signal producing 91% sparse reconstructed signal.

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