International Journal of Recent Trends in Engineering & Research

ISSN (ONLINE) : 2455 – 1457

IMPACT FACTOR : 4.101

Somewhat Compactness and Somewhat Connectedness in Topological Spaces

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Abstract: In 2016 Baker introduced the notion of somewhat open set in topological space and used it to characterize somewhat continuity and contra – somewhat continuity. In this paper we originally originate the notion of somewhat – compact space and interpret its several effects and characterizations. Also we newly originate and study the concepts of somewhat – Lindelof spaces and somewhat connected spaces.

Key Words and Phrases: Topological Space, Somewhat – Open Set, Somewhat Lindelof Space, Somewhat Connected space.

2010 Mathematics Subject Classification: 54B05, 54C20, 54D30.

I. INTRODUCTION

Baker introduced the notion of somewhat open set in topological space in 2016 and used it to characterize somewhat continuity and contra – somewhat continuity. Throughout this paper, (X,τ) and (Y,σ) (or simply, X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X,τ), $Cl(A)$, $Int(A)$ and $X-A$ denote the closure of A, the interior of A and the complement of A in X, respectively. Now we bring up with the new concepts of somewhat – compact, somewhat – Lindelof, Countably somewhat – compact and somewhat connected spaces and investigate several properties and characterizations for these concepts.

II. PRELIMINARIES

DEFINITION 2.1. A subset $\mathbf{U}$ of a space X is said to be somewhat open if U = ∅ or if there exist $\mathbf{x\in U}$ and an open subset V such that $\mathbf{x\in V\subseteq U}$. A set is called somewhat closed if its complement is somewhat open.

Obviously somewhat open sets are closed under arbitrary union but, as we see in the following example, not closed under intersection. The set of all somewhat open sets in a topological space $(\mathbf{X},\tau)$ is denoted by $\mathbf{SW}(\mathbf{X},\tau)$. Clearly $\tau\subseteq\mathbf{SW}(\mathbf{X},\tau)$. Also any set containing a nonempty somewhat open set is somewhat open. Semi-open implies somewhat open and the closure of a preopen set is somewhat open.

EXAMPLE 2.2. Let X = {a, b, c} have the topology τ = {X, ∅, {a}, {b}, {a, b}}. The sets {a, c} and {b, c} are somewhat open, but, since {c} is not somewhat open, somewhat open sets are not closed under intersection.

DEFINITION 2.3. Let $\mathbf{A}$ be a subset of a space X. The somewhat closure of A, denoted by $\mathbf{swCl}(\mathbf{A})$, is given by $\mathbf{swCl}(\mathbf{A})=\bigcap\{\mathbf{F}:\mathbf{F}\text{ is somewhat closed and }\mathbf{A}\subseteq\mathbf{F}\}$ and the somewhat interior of $\mathbf{A}$, denoted by $\mathbf{swInt}(\mathbf{A})$, is given by $\mathbf{swInt}(\mathbf{A})=\bigcup\{\mathbf{U}\subseteq\mathbf{A}:\mathbf{U}\text{ is somewhat open}\}$.

The following properties of the closure and interior operators for somewhat open sets are stated for completeness. They are special cases of properties of operators defined for minimal structures by Popa and Noiri [36].

THEOREM 2.4. The following statements hold for a subset A of a space X:

(a) $\mathbf{swInt(X-A)=X-swCl(A).}$

(b) $\mathbf{swCl(X-A)=X-swInt(A).}$

(c) $\mathbf{swCl(A)}$ is somewhat closed.

(d) A is somewhat closed if and only if $\mathbf{swCl(A)=A.}$

(e) $\mathbf{swCl(A)=\{x\in X: \text{for every somewhat open subset U containing x, } U\cap A\neq\phi\}.}$

THEOREM 2.5. Let A be a subset of a space X. Then $\mathbf{swCl(A)=X}$ if A is dense in X, and $\mathbf{swCl(A)=A}$ if A is not dense in X.

PROOF: Assume A is dense in X. Let x ∈ X and let U be a somewhat open set containing x. Then there exists a nonempty open set V such that x ∈ V ⊆ U. Since A is dense in X, $\mathbf{A\cap V\neq\phi.}$ Then $\mathbf{A\cap U\neq\phi}$ and hence x ∈ swCl(A), which shows that swCl(A) = X. Assume A is not dense in X. Let x ∈ X such that $\mathbf{x\notin Cl(A).}$ Then there exists an open set U such that x ∈ U and $\mathbf{U\cap A=\phi.}$ Thus $\mathbf{x\in U\subseteq(X-A),}$ which proves that X − A is somewhat open and that A is somewhat closed. Thus $\mathbf{swCl(A)=A.}$

COROLLARY 2.6. Let A be a subset of a space X. Then swInt(A) = A if X − A is not dense in X, and swInt(A) = ∅ if X − A is dense in X.

THEOREM 2.7. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be a function. Then the following statements are equivalent:

(a) f is somewhat continuous.

(b) For every open subset V of Y, $f^{-1}(V)$ is somewhat open.

(c) For every closed subset F of Y, $f^{-1}(F)$ is somewhat closed.

(d) For every x ∈ X and every open subset V of Y containing $f(x)$ there exists a somewhat open set U of X containing x such that $f(U)\subseteq V.$

DEFINITION 2.8. Let (X, τ) and (Y, σ) be two topological spaces. A function $f:(X,\tau)\longrightarrow(Y,\sigma)$ is called somewhat irresolute if $f^{-1}(V)$ is somewhat closed in (X, τ) for every somewhat closed set V of (Y, σ).

DEFINITION 2.9. Let (X, τ) and (Y, σ) be two topological spaces. A function $f:(X,\tau)\longrightarrow(Y,\sigma)$ is called strongly somewhat continuous if the inverse image of every somewhat closed in Y is closed in X.

DEFINITION 2.10. Let (X, τ) and (Y, σ) be two topological spaces. A function $f:(X,\tau)\longrightarrow(Y,\sigma)$ is called perfectly somewhat continuous if the inverse image of every somewhat closed set in Y is both closed and open set in X.

III. SOMEWHAT COMPACTNESS

DEFINITION 3.1. A collection $\{A_i : i \in I\}$ of somewhat open sets in a topological space (X, τ)) is called a somewhat-open cover of a subset B of X if $B \subseteq \cup\{A_i : i \in I\}$ holds.

DEFINITION 3.2. A topological space (X, τ) is somewhat compact if every somewhat open cover of X has a finite subcover.

DEFINITION 3.3. A subset B of a topological space (X, τ) is said to be somewhat compact relative to (X, τ) if, for every collection $\{A_i : i \in I\}$ of somewhat open subsets of X such that $B \subseteq \cup\{A_i : i \in I\}$ there exists a finite subset I_0 of I such that $B \subseteq \cup\{A_i : i \in I_0\}$.

DEFINITION 3.4. A subset B of a topological space (X, τ) is said to be somewhat compact if B is somewhat compact as a subspace of X.

THEOREM 3.5. Every somewhat compact space is compact.
PROOF. Let $\{A_i : i \in I\}$ be an open cover of (X, τ). Since $\tau \subseteq SW(X, \tau)$. So $\{A_i : i \in I\}$ is a somewhat open cover of (X, τ). Since (X, τ) is somewhat compact. So somewhat open cover $\{A_i : i \in I\}$ of (X, τ) has a finite subcover say $\{A_i : i = 1, 2, ..., n\}$ for X. Hence (X, τ) is a somewhat compact space.

THEOREM 3.6. Every somewhat closed subset of a somewhat compact space is somewhat compact relative to X.
PROOF. Let A be a somewhat closed subset of a topological space (X, τ). Then $A^C = X - A$ is somewhat open in (X, τ). Let $\gamma = \{A_i : i \in I\}$ be a somewhat open cover of A by somewhat open subsets in (X, τ). Let $\gamma^* = \{A_i : i \in I\} \cup \{A^C\}$ be a somewhat open cover of (X, τ). That is $X = \cup\gamma^* = (\cup\{A_i : i \in I\}) \cup A^C$. By hypothesis (X, τ) is somewhat compact and hence γ^* is reducible to a finite sub cover of (X, τ) say $X = A_{i_1} \cup A_{i_2} \cup ... \cup A_{i_n} \cup A^C$, $A_{i_k} \in \gamma$. But A and A^C are disjoint. Hence $A \subseteq A_{i_1} \cup A_{i_2} \cup ... \cup A_{i_n}$, $A_{i_k} \in \gamma$. Thus a somewhat open cover γ of A contains a finite subcover. Hence A is somewhat compact relative to (X, τ).

THEOREM 3.7 A somewhat continuous image of a somewhat compact space is compact.
PROOF. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a somewhat continuous map from a somewhat compact space X onto a topological space Y. Let $\{A_i : i \in I\}$ be an open cover of Y. Then $f^{-1}(\{A_i : i \in I\})$ is a somewhat open cover of X, as f is somewhat continuous. Since X is somewhat compact, the somewhat open cover of X, $f^{-1}(\{A_i : i \in I\})$ has a finite sub cover say $\{f^{-1}(A_i) : i = 1, 2, ..., n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1, 2, ..., n\}$, which implies $f(X) = \cup\{A_i : i = 1, 2, ..., n\}$, then $Y = \cup\{A_i : i = 1, 2, ..., n\}$. That is $\{A_i : i = 1, 2, ..., n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for Y. Hence Y is compact.

THEOREM 3.8. If a map $f:(X,\tau)\longrightarrow(Y,\sigma)$ is somewhat - irresolute and a subset S of X is somewhat compact relative to (X, τ), then the image $f(S)$ is somewhat compact relative to (Y, σ).
PROOF. Let $\{A_i : i\in I\}$ be a collection of somewhat open cover of (Y, σ), such that $f(S)\subseteq \cup\{A_i : i\in I\}$. Then $S\subseteq \cup\{f^{-1}(A_i): i\in I\}$, where $\{f^{-1}(A_i): i\in I\}\subseteq SW(X,\tau)$. Since S is somewhat compact relative to (X, τ), there exists a finite subcollection $\{A_1, A_2, ..., A_n\}$ such that $S\subseteq \cup\{f^{-1}(A_i): i=1,2,...,n\}$. That is $f(S)\subseteq \cup\{A_1, A_2, ..., A_n\}$. Hence $f(S)$ is somewhat compact relative to (Y, σ).

THEOREM 3.9. If a map $f:(X,\tau)\longrightarrow(Y,\sigma)$ is strongly somewhat continuous map from a compact space (X, τ) onto a topological space (Y, σ), then (Y, σ) is somewhat compact.
PROOF. Let $\{A_i : i\in I\}$ be a somewhat open cover of (Y, σ). Since f is strongly somewhat continuous, $\{f^{-1}(A_i): i\in I\}$ is an open cover of (X, τ). Again, since (X, τ) is compact, the open cover $\{f^{-1}(A_i): i\in I\}$ of (X, τ) has a finite sub cover say $\{f^{-1}(A_i): i=1,2,...,n\}$. Therefore $X=\cup\{f^{-1}(A_i): i=1,2,...,n\}$, which implies $f(X)=\cup\{A_i : i=1,2,...,n\}$, so that $Y=\cup\{A_i : i=1,2,...,n\}$. That is $A_1, A_2, ..., A_n$ is a finite sub cover of $\{A_i : i\in I\}$ for (Y, σ). Hence (Y, σ) is somewhat compact.

THEOREM 3.10. If a map $f:(X,\tau)\longrightarrow(Y,\sigma)$ is perfectly somewhat continuous map from a compact space (X, τ) onto a topological space (Y, σ), then (Y, σ) is somewhat compact.
PROOF. Let $\{A_i : i\in I\}$ be a somewhat open cover of (Y, σ). Since f is perfectly somewhat continuous, $\{f^{-1}(A_i): i\in I\}$ is an open cover of (X, τ). Again, since (X, τ) is compact, the open cover $\{f^{-1}(A_i): i\in I\}$ of (X, τ) has a finite sub cover say $\{f^{-1}(A_i): i=1,2,...,n\}$. Therefore $X=\cup\{f^{-1}(A_i): i=1,2,...,n\}$, which implies $f(X)=\cup\{A_i : i=1,2,...,n\}$, so that $Y=\cup\{A_i : i=1,2,...,n\}$. That is $\{A_1, A_2, ..., A_n\}$ is a finite sub cover of $\{A_i : i\in I\}$ for (Y, σ). Hence (Y, σ) is somewhat compact.

THEOREM 3.11. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be somewhat irresolute map from somewhat compact space (X, τ) onto topological space (Y, σ), then (Y, σ) is somewhat compact.
PROOF. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be somewhat irresolute map from a somewhat compact space (X, τ) onto a topological space (Y, σ). Let $\{A_i : i\in I\}$ be a somewhat open cover of (Y, σ). Then $\{f^{-1}(A_i): i\in I\}$ is a somewhat open open cover of (X, τ), since f is somewhat irresolute. As (X,τ) is somewhat compact, the somewhat open cover $\{f^{-1}(A_i): i\in I\}$ of (X, τ) has a finite subcover say $\{f^{-1}(A_i): i=1,2,...,n\}$. Therefore $X=\cup\{f^{-1}(A_i): i=1,2,...,n\}$, which implies $f(X)=\cup\{A_i : i=1,2,...,n\}$, so that $Y=\cup\{A_i : i=1,2,...,n\}$. That is $\{A_1, A_2, ..., A_n\}$ is a finite sub cover of $\{A_i : i\in I\}$ for (Y, σ). Hence (Y, σ) is somewhat compact.

THEOREM 3.12. If (X,τ) is compact and every somewhat closed set in X is also cloed in X, then (X,τ) is somewhat compact.

PROOF. Let $\{A_i : i \in I\}$ be a somewhat open cover of X. Since every somewhat closed set in X is also closed in X. Thus clearly $\{A_i : i \in I\}$ is also an open cover of X. Since (X,τ) is compact. So there exists a finite subcover $\{A_i : i = 1,2,\ldots,n\}$ of $\{A_i : i \in I\}$ such that $X = \cup\{A_i : i = 1,2,\ldots,n\}$. Hence (X, τ) is a somewhat compact space.

THEOREM 3.13. A topological space (X, τ) is somewhat compact if and only if every family of somewhat closed sets of (X, τ) having finite intersection property has a nonempty intersection.

PROOF. Suppose (X, τ) is somewhat compact. Let $\{A_i : i \in I\}$ be a family of somewhat closed sets with finite intersection property. Suppose $\cap\{A_i : i \in I\} = \phi$, then $X - (\cap\{A_i : i \in I\}) = X$. This implies $\cup\{(X - A_i) : i \in I\} = X$. Thus the cover $\{(X - A_i) : i \in I\}$ is a somewhat open cover of (X,τ). Then, the somewhat open cover $\{(X - A_i) : i \in I\}$ has a finite sub cover say $\{(X - A_i) : i = 1,2,\ldots,n\}$. This implies $X = \cup\{(X - A_i) : i = 1,2,\ldots,n\}$ which implies $X = X - \cap\{A_i : i = 1,2,\ldots,n\}$, which implies $X - X = [\cap\{A_i : i = 1,2,\ldots,n\}]$ which implies $\phi = \cap\{A_i : i = 1,2,\ldots,n\}$. This disproves the assumption. Hence $\cap\{A_i : i = 1,2,\ldots,n\} \neq \phi$.

Conversely suppose (X, τ) is not somewhat compact. Then there exits a somewhat open cover of (X, τ) say $\{G_i : i \in I\}$ having no finite subcover. This implies that for any finite sub family $\{G_i : i = 1,2,\ldots,n\}$ of $\{G_i : i \in I\}$, we have $\cup\{G_i : i = 1,2,\ldots,n\} \neq X$, which implies $X - (\cup\{G_i : i = 1,2,\ldots,n\}) \neq X - X$, therefore $\cap\{G_i : i = 1,2,\ldots,n\} \neq \phi$. Then the family $\{X - G_i : i \in I\}$ of somewhat closed sets has a finite intersection property. Also by assumption $\cap\{X - G_i : i = 1,2,\ldots,n\} \neq \phi$ which implies $X - (\cup\{G_i : i = 1,2,\ldots,n\}) \neq \phi$, so that $\cup\{G_i : i = 1,2,\ldots,n\} \neq X$. This implies $\{G_i : i \in I\}$ is not a cover of (X, τ). This disproves the fact that $\{G_i : i \in I\}$ is a cover for (X, τ). Therefore a somewhat open cover $\{G_i : i \in I\}$ of (X, τ) has a finite subcover $\{G_i : i = 1,2,\ldots,n\}$. Hence (X, τ) is somewhat compact.

THEOREM 3.14. Let A be a somewhat compact set relative to a topological space X and B be a somewhat closed subset of X. Then A∩ B is somewhat compact relative to X.

PROOF. Let A be somewhat compact relative to X. Suppose that $\{A_i : i \in I\}$ is a cover of A∩ B by somewhat open sets in X. Then $\{A_i : i \in I\} \cup \{B^C\}$ is a cover of A by somewhat open sets in X, but A is somewhat compact relative to X, so there exist $i_1, i_2, \ldots, i_n$ such that $A \subseteq \left(\cup\left\{A_{i_j} : j = 1,2,\ldots,n\right\}\right) \cup B^C$. Then $A \cap B \subseteq \cup\left\{\cup A_{i_j} \cap B : j = 1,2,\ldots,n\right\} \subseteq \cup\left\{A_{i_j} : j = 1,2,\ldots,n\right\}$. Hence A∩ B is somewhat compact relative to X.

THEOREM 3.15. If a function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is somewhat irresolute and a subset of X is somewhat compact relative to X, then $f(B)$ is somewhat compact relative to Y.

PROOF. Let $\{A_i : i \in I\}$ be a cover of $f(B)$ by somewhat open subsets of Y. Since f is somewhat irresolute. Then $\{f^{-1}(A_i) : i \in I\}$ is a cover of B by somewhat open subsets of X. Since B is somewhat compact relative to X, $\{f^{-1}(A_i) : i \in I\}$ has a finite subcover say $\{f^{-1}(A_1), f^{-1}(A_2), ..., f^{-1}(A_n)\}$ for B. Now $\{A_i : i = 1,2,...,n\}$ is a finite subcover of $\{A_i : i \in I\}$ for $f(B)$. So $f(B)$ is somewhat compact relative to Y.

IV. COUNTABLY SOMEWHAT COMPACTNESS

In this section, we concentrate on the concept of countably somewhat compactness and their properties.

DEFINITION 4.1. A topological space (X, τ) is said to be countably somewhat compact if every countable somewhat open cover of X has a finite subcover.

THEOREM 4.2. If (X, τ) is a countably somewhat compact space, then (X, τ) is countably compact.

PROOF. Let (X, τ) be countably somewhat compact space. Let $\{A_i : i \in I\}$ be a countable open cover of (X, τ). Since $\tau \subseteq SW(X, \tau)$. So $\{A_i : i \in I\}$ is a countable somewhat open cover of (X, τ). Since (X, τ) is countably somewhat compact, countable somewhat open cover $\{A_i : i \in I\}$ of (X, τ) has a finite subcover say $\{A_i : i = 1,2,...,n\}$ for X. Hence (X, τ) is a countably compact space.

THEOREM 4.2. If (X, τ) is countably compact and every somewhat closed subset of X is closed in X, then (X, τ) is countably somewhat compact.

PROOF. Let (X, τ) be countably compact space. Let $\{A_i : i \in I\}$ be a countable somewhat open cover of (X, τ). Since every somewhat closed subset of X is closed in X. Thus every somewhat open set in X is open in X, Therefore $\{A_i : i \in I\}$ is a countable open cover of (X, τ). Since (X, τ) is countably compact, countable open cover $\{A_i : i \in I\}$ of (X, τ) has a finite sub cover say $\{A_i : i = 1,2,...,n\}$ for X. Hence (X, τ) is a countably somewhat compact space.

THEOREM 4.3. Every somewhat compact space is countably somewhat compact.

PROOF. Let (X, τ) be somewhat compact space. Let $\{A_i : i \in I\}$ be a countable somewhat open cover of (X, τ). Since (X, τ) is somewhat compact, somewhat pen cover $\{A_i : i \in I\}$ of (X, τ) has a finite subcover say $\{A_i : i = 1,2,...,n\}$ for (X, τ). Hence (X, τ) is a countably somewhat compact space.

THEOREM 4.4. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a somewhat continuous injective mapping. If X is countably somewhat compact space, then (Y, σ) is countably compact.

PROOF. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a somewhat continuous map from a countably somewhat compact (X, τ) onto a topological space (Y, σ). Let $\{A_i : i \in I\}$ be a countable open cover of Y. Then $\{f^{-1}(A_i) : i \in I\}$ is a countable somewhat open cover of X, as f is somewhat continuous. Since X is countably somewhat compact, the countable somewhat open cover $\{f^{-1}(A_i) : i \in I\}$ of X has a finite sub cover say $\{f^{-1}(A_i) : i = 1,2,...,n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1,2,...,n\}$, which implies $f(X) = \mathrm{U}\{A_i : i = 1, 2, \ldots, n\}$, then $Y = \mathrm{U}\{A_i : i = 1, 2, \ldots, n\}$. That is $\{A_i : i = 1, 2, \ldots, n\}$ is a finite subcover of $\{\mathrm{A_i} : \mathrm{i} \in \mathrm{I}\}$ for Y. Hence Y is countably compact.

THEOREM 4.5. If a map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is perfectly somewhat continuous map from a countably compact space (X, τ) onto a topological space (Y, σ), then (Y, σ) is countably somewhat compact.
PROOF. Let $\{\mathrm{A_i} : \mathrm{i} \in \mathrm{I}\}$ be a countable somewhat open cover of (Y, σ). Since f is perfectly somewhat continuous, $\{f^{-1}(A_i) : i \in I\}$ is a countable open cover of (X, τ). Again, since (X, τ) is countably compact, the countable open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ) has a finite subcover say $\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$. Therefore $X = \mathrm{U}\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$, which implies $f(X) = \mathrm{U}\{A_i : i = 1, 2, \ldots, n\}$, so that $Y = \mathrm{U}\{A_i : i = 1, 2, \ldots, n\}$. That is $\{\mathrm{A_1}, \mathrm{A_2}, \ldots, \mathrm{A_n}\}$ is a finite subcover of $\{\mathrm{A_i} : \mathrm{i} \in \mathrm{I}\}$ for (Y, σ). Hence (Y, σ) is countably somewhat compact.

THEOREM 4.6. If a map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is strongly somewhat continuous map from a countably compact space (X, τ) onto a topological space (Y, σ), then (Y, σ) is countably somewhat compact.
PROOF. Let $\{\mathrm{A_i} : \mathrm{i} \in \mathrm{I}\}$ be a countable somewhat open cover of (Y, σ). Since f is strongly somewhat continuous, $\{f^{-1}(A_i) : i \in I\}$ is a countable open cover of (X, τ). Again, since (X, τ) is countably compact, the countable open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$. Therefore $X = \mathrm{U}\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$, which implies $f(X) = \mathrm{U}\{A_i : i = 1, 2, \ldots, n\}$, so that $Y = \mathrm{U}\{A_i : i = 1, 2, \ldots, n\}$. That is $\{\mathrm{A_1}, \mathrm{A_2}, \ldots, \mathrm{A_n}\}$ is a finite subcover of $\{\mathrm{A_i} : \mathrm{i} \in \mathrm{I}\}$ for (Y, σ). Hence (Y,σ) is countably somewhat compact.

Theorem 4.7. The image of a countably somewhat compact space under a somewhat irresolute map is countably somewhat compact.
PROOF. If a map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is somewhat irresolute map from a countably somewhat compact space (X, τ) onto a topological space (Y, σ). Let $\{\mathrm{A_i} : \mathrm{i} \in \mathrm{I}\}$ be a countable somewhat open cover of (Y, σ). Then $\{f^{-1}(A_i) : i \in I\}$ is a countable somewhat open cover of (X, τ), since f is somewhat irresolute. As (X, τ) is countably somewhat compact, the countable somewhat open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ) has a finite subcover say $\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$. Therefore $X = \mathrm{U}\{f^{-1}(A_i) : i = 1, 2, \ldots, n\}$, which implies $f(X) = \mathrm{U}\{A_i : i = 1, 2, \ldots, n\}$, so that $Y = \mathrm{U}\{A_i : i = 1, 2, \ldots, n\}$. That is $\{\mathrm{A_1}, \mathrm{A_2}, \ldots, \mathrm{A_n}\}$ is a finite subcover of $\{\mathrm{A_i} : \mathrm{i} \in \mathrm{I}\}$ for (Y, σ). Hence (Y, σ) is countably somewhat compact.

V. SOMEWHAT LINDELOF SPACE

In this section, we concentrate on the concept of somewhat Lindelof space and their properties.

DEFINITION 5.1. A topological space (X, τ) is said to be somewhat Lindelof space if every somewhat open cover of X has a countable subcover.

THEOREM 5.2. Every somewhat Lindelof space is Lindelof space.
PROOF. Let $\{A_i : i \in I\}$ be an open cover of (X, τ). Since $\tau \subseteq SW(X,\tau)$. Therefore $\{A_i : i \in I\}$ is a somewhat open cover of (X, τ). Since (X, τ) is somewhat Lindelof space, somewhat open cover $\{A_i : i \in I\}$ of (X, τ) has a countable subcover say $\{A_i : i \in J\}$ for X, for some countable subset J of I. Hence (X, τ) is a Lindelof space.

THEOREM 5.3. If (X, τ) is Lindelof space and every somewhat closed subset of X is closed in X, then (X, τ) is somewhat Lindelof space.
PROOF. Let $\{A_i : i \in I\}$ be a somewhat open cover of (X, τ). Since every somewhat closed subset of X is closed in X. So every somewhat open set in X is open in X. Therefore $\{A_i : i \in I\}$ is an open cover of (X, τ). Since (X, τ) is compact, open cover $\{A_i : i \in I\}$ of (X, τ) has a countable subcover say $\{A_i : i \in J\}$ for X, for some countable subset J of I. Hence (X, τ) is a somewhat Lindelof space.

THEOREM 5.4. Every somewhat compact space is somewhat Lindelof space.
PROOF. Let $\{A_i : i \in I\}$ be a somewhat open cover of (X, τ). Since (X, τ) is somewhat compact space. Then $\{A_i : i \in I\}$ has a finite subcover say $\{A_i : i = 1,2,...,n\}$. Since every finite subcover is always countable subcover and therefore $\{A_i : i = 1,2,...,n\}$ is countable subcover of $\{A_i : i \in I\}$. Hence (X, τ) is somewhat Lindelof space.

THEOREM 5.5. A somewhat continuous image of a somewhat Lindelof space is Lindelof space.
PROOF. Let $f : (X,\tau) \longrightarrow (Y,\sigma)$ be a somewhat continuous map from a somewhat Lindelof space X onto a topological space Y. Let $\{A_i : i \in I\}$ be an open cover of Y. Then $\{f^{-1}(A_i) : i \in I\}$ is a somewhat open cover of X, as f is somewhat continuous. Since X is somewhat Lindelof space, the somewhat open cover $\{f^{-1}(A_i) : i \in I\}$ of X has a countable subcover say $\{f^{-1}(A_i) : i = 1,2,3,....\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1,2,3,....\}$, which implies $f(X) = \cup\{A_i : i = 1,2,3,....\}$, then $Y = \cup\{A_i : i = 1,2,3,....\}$. That is $\{A_1, A_2, A_3, ...\}$ is a countable subcover of $\{A_i : i \in I\}$ for Y. Hence Y is Lindelof space.

THEOREM 5.6. The image of a somewhat Lindelof space under a somewhat irresolute map is somewhat Lindelof space.
PROOF. Let a map $f : (X,\tau) \longrightarrow (Y,\sigma)$ be somewhat irresolute map from a somewhat Lindelof space (X, τ) onto a topological space (Y, σ). Let $\{A_i : i \in I\}$ be a somewhat open cover of (Y, σ). Then $\{f^{-1}(A_i) : i \in I\}$ is a somewhat open cover of (X, τ). Since f is somewhat irresolute. As (X,τ) is somewhat Lindelof space, the somewhat open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ) has a countable subcover say $\{f^{-1}(A_{i_n}) : n \in N\}$. Therefore $X = \cup\{f^{-1}(A_{i_n}) : n \in N\}$, which implies $X = \cup\{f^{-1}(A_{i_n}) : n \in N\}$, then $f(X) = \cup\{A_{i_n} : n \in N\}$, so that $Y = \cup\{A_{i_n} : n \in N\}$. That is $\{A_{i_n} : n \in N\}$ is a countable subcover of $\{A_i : i \in I\}$ for (Y, σ). Hence (Y, σ) is somewhat Lindelof space.

THEOREM 5.7. If (X,τ) is somewhat Lindelof space and countably somewhat compact space, then (X, τ) is somewhat compact space.
PROOF. Suppose (X, τ) is somewhat Lindelof space and countably somewhat compact space. Let $\{A_i : i \in I\}$ be a somewhat open cover of (X, τ). Since (X, τ) is somewhat Lindelof space $\{A_i : i \in I\}$ has a countable sub cover say $\{A_{i_n} : n \in N\}$, therefore $\{A_{i_n} : n \in N\}$, is a countable subcover of (X,τ) and $\{A_{i_n} : n \in N\}$, is a subfamily of $\{A_i : i \in I\}$ and so $\{A_{i_n} : n \in N\}$ is a countable somewhat open cover of (X, τ). Again, since (X, τ) is countably somewhat compact, $\{A_{i_n} : n \in N\}$ has a finite subcover and say $\{A_{i_k} : k = 1,2,\ldots,n\}$. Therefore $\{A_{i_k} : k = 1,2,\ldots,n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (X, τ). Hence (X, τ) is somewhat compact space.

THEOREM 5.8. If a function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is somewhat irresolute and a subset of X is somewhat Lindelof relative to X, then $f(B)$ is somewhat Lindelof relative to Y.
PROOF. Let $\{A_i : i \in I\}$ be a cover of $f(B)$ by somewhat open subsets of Y. Then $\{f^{-1}(A_i) : i \in I\}$ is a cover of B by somewhat open subsets of X. Since B is somewhat Lindelof relative to X, $\{f^{-1}(A_i) : i \in I\}$ has a countable subcover say $\{f^{-1}(A_1), f^{-1}(A_2), f^{-1}(A_3), \ldots.\}$ for B. Now $\{A_1, A_2, A_3, \ldots.\}$ is a countable sub cover of $\{A_i : i \in I\}$ for $f(B)$. So $f(B)$ is somewhat Lindelof relative to Y.

VI. SOMEWHAT CONNECTEDNESS

DEFINITION 6.1. A topological space (X, μ) is said to be somewhat Connected if X cannot be written as a disjoint union of two nonempty somewhat open sets. A subset of (X, μ) is somewhat connected if it is somewhat connected as a subspace.

THEOREM 6.2. Every somewhat connected space is connected.
PROOF. Let A and B be two nonempty disjoint proper open sets in X. Since every open set is somewhat open set. Therefore A and B are nonempty disjoint proper somewhat open sets in X and X is somewhat connected space. Therefore $X \neq A \cup B$. Therefore X is connected.

EXAMPLE 6.3. Let X = {a, b, c} and τ = {X, φ, {a}}. Then it is somewhat connected.

REMARK 6.4. The converse of the above theorem need not be true in general, which follows from the following example.

EXAMPLE 6.5. Let X = {a, b, c} and τ = {X, φ, {a}, {b}, {a, b}}. Clearly (X, τ) is connected. The somewhat open sets of X are given by $SW(X,\tau) =$ {X, φ, {b, c}, {a, c}, {a, b}, {b}, {a}}. Therefore (X, τ) is not a somewhat connected space, since X = {b, c}∪{a} where {b, c} and {a} are nonempty somewhat open sets.

THEOREM 6.6. For a topological space (X, τ) the following are equivalent
(i) (X, τ) is somewhat connected.

(ii) The only subsets of (X, τ) which are both somewhat open and somewhat closed are the empty set X and ϕ.
(iii) Each somewhat continuous map of (X, τ) into a discrete space (Y, σ) with at least two points is a constant map.
PROOF. (1)⇒(2) Let G be a somewhat open and somewhat closed subset of (X, τ). Then $X-G$ is also both somewhat open and somewhat closed. Then $X = G \cup (X-G)$ is a disjoint union of two nonempty somewhat open sets which contradicts the fact that (X, τ) is somewhat connected. Hence $G=\phi$ or $G=X.$
(2)⇒(1) Suppose that $X = A \cup B$ where A and B are disjoint nonempty somewhat open subsets of (X, τ). Since $A = X-B,$ then A is both somewhat open and somewhat closed. By assumption $A=\phi$ or $A = X,$ which is a contradiction. Hence (X, τ) is somewhat connected.
(2)⇒(3) Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be a somewhat continuous map, where (Y, σ) is discrete space with at least two points. Then $f^{-1}(y)$ is somewhat closed and somewhat open for each y∈ Y. That is (X, τ) is covered by somewhat closed and somewhat open covering $\{f^{-1}(y): y\in Y\}.$ By assumption, $f^{-1}(y)=\phi$ or $f^{-1}(y)=X$ for each y∈ Y. If $f^{-1}(y)=\phi$ for each y ∈ Y, then f fails to be a map. Therefore their exists at least one point say $f^{-1}(y^*)\neq\phi,$ $y^*\in Y$ such that $f^{-1}(y^*)=X.$ This shows that f is a constant map.
(3)⇒(2) Let G be both somewhat open and somewhat closed in (X, τ). Suppose $G\neq\phi.$ Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be a somewhat continuous map defined by $f(G)=\{a\}$ and $f(X-G)=\{b\}$ where $a\neq b$ and a, b ∈ Y. By assumption, f is constant so G = X.

THEOREM 6.7. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be a somewhat continuous surjection and (X, τ) is somewhat connected. Then (Y, σ) is connected .
PROOF. Suppose (Y, σ) is not connected. Let Y = A∪ B, where A and B are disjoint nonempty open subsets in (Y, σ). Since f is somewhat continuous, $X = f^{-1}(A)\cup f^{-1}(B),$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty somewhat open subsets in (X, τ). This disproves the fact that (X, τ) is somewhat connected. Hence (Y, σ) is connected.

THEOREM 6.8. If $f:(X,\tau)\longrightarrow(Y,\sigma)$ is a somewhat irresolute surjection and X is somewhat connected, then Y is somewhat connected.
PROOF. Suppose that Y is not somewhat connected. Let Y = A∪B, where A and B are nonempty somewhat open sets in Y. Since f is somewhat irresolute and onto, $X = f^{-1}(A)\cup f^{-1}(B),$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty somewhat open sets in (X, τ). This contradicts the fact that (X, τ) is somewhat connected. Hence (Y, σ) is somewhat connected.

THEOREM 6.9. Suppose that every somewhat closed set in X is closed in X and X is connected. Then X is somewhat connected.
PROOF. Suppose that X is connected. Then X cannot be expressed as disjoint union of two nonempty proper open subset of X. Suppose X is not somewhat connected space. Let A and B be any two nonempty somewhat open subsets of X such that X = A∪ B, where A∩B = φ and $A\subseteq X,$ $B\subseteq X.$ Since every somewhat closed set in X is closed in X. So every somewhat open set in X is open in X. Hence A, B are open subsets of X, which contradicts that X is connected. Therefore X is somewhat connected.

THEOREM 6.10. If the somewhat open sets C and D form a separation of X and if Y is somewhat connected subspace of X, then Y lies entirely within C or D.

PROOF. By hypothesis C and D are both somewhat open in X. The sets C∩Y and D∩Y are somewhat open in Y, these two sets are disjoint and their union is Y. If they were both nonempty, they would constitute a separation of Y. Therefore, one of them is empty. Hence Y must lie entirely in C or D.

THEOREM 6.11. Let A be a somewhat connected subspace of X. If $A \subseteq B \subseteq swCl(A)$, then B is also somewhat connected.

PROOF. Let A be somewhat connected. Let $A \subseteq B \subseteq swCl(A)$. Suppose that B = C∪D is a separation of B by somewhat open sets. Thus by previous theorem above A must lie entirely in C or D. Suppose that $A \subseteq C$, then $swCl(A) \subseteq swCl(C)$. Since $swCl(C)$ and D are disjoint, B cannot intersect D. This disproves the fact that D is non empty subset of B. So $D = \phi$ which implies B is somewhat connected.

ACKNOWLEDGEMENT

The author is highly and gratefully indebted to the Prince Mohammad Bin Fahd University, Al Khobar, Saudi Arabia, for providing necessary research facilities during the preparation of this research paper.

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