

Estimation of Non-Parametric Function Using Wavelet Transform

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Abstract: The Wavelet has number of functions for the estimation of an unidentified function (signal or image) in noise. This paper provides non-parametric function estimation of denoise signals/images. Non-parametric function estimation aims to estimate or recover or denoise a function of interest, perhaps a signal, spectrum or image, that is observed in noise and possibly indirectly after some transformation, as in deconvolution. 'Non-parametric' signifies that no a priori limit is placed on the number of unknown parameters used to model the signal. Such theories of estimation are necessarily quite different from traditional statistical models with a small number of parameters specified in advance. Before wavelets, the theory was dominated by linear estimators, and the exploitation of assumed smoothness in the unknown function to describe optimal methods. Wavelets provide a set of tools that make it natural to assert, in plausible theoretical models, that the sparsity of representation is a more basic notion than smoothness, and that nonlinear thresholding can be a powerful competitor to traditional linear methods. Wavelet analysis has been found to be a powerful tool for the nonparametric estimation of spatially-variable objects. We discuss in detail wavelet methods in nonparametric regression, where the data are modelled as observations of a signal contaminated with additive Gaussian noise, and provide an extensive review of the massive literature of wavelet shrinkage and wavelet thresholding estimators developed to denoise such data. These estimators arise from a wide range of classical and empirical Bayes methods treating either individual or blocks of wavelet coefficients. Compare various estimators in a wide-ranging simulation study on a variety of sample sizes, test functions, signal-to-noise ratios and wavelet filters.

Keywords: Sparsity, denoise, statistical, wavelets

I. INTRODUCTION

Denosing is the process, which reconstruct a signal from a noisy one. The wavelet transform (WT) a powerful tool of signal and image processing that have been successfully used in many scientific fields such as signal processing, image compression, computer graphics, and pattern recognition. The classical Fourier Transform, the WT is particularly suitable for the applications of non-stationary signals which may instantaneous vary in time. It is crucial to analyze the time-frequency characteristics of the signals which classified as non-stationary or transient signals in order to understand the exact features of such signals. For this purpose, primarily, researchers has focused on continuous wavelet transform (CWT) that gives more reliable and detailed time-scale representation rather than the classical short time Fourier transform (STFT) giving a time-frequency representation. The wavelet coefficients represent a

measure of similarity in the frequency content between a signal and a chosen wavelet function. These coefficients are computed as a convolution of the signal and the scaled wavelet function, which can be interpreted as a dilated band-pass filter because of its band-pass like spectrum. The CWT technique expands the signal onto basis functions created by expanding, shrinking and shifting a single prototype function, which named as mother wavelet, specially selected for the signal under considerations. A mother wavelet has satisfy that it has a zero mean value, which require that the transformation kernel of the wavelet transform compactly supports localization in time, thereby offering the potential to capture the spikes occurring instantly in a short period of time. A wavelet expansion is representation of a signal in terms of an orthogonal collection of real-valued generated by applying suitable transformation to the original selected wavelet. The properties and advantages of a wavelet families based upon the mother wavelet features. The expansion is formed by two dimensional expansion of a signal and thus provides a time-frequency localization of the input signal. This suggests that most of the energy of the signal will be captured an insufficient coefficient. The basic functions in a wavelet transform are produced from the mother wavelet by scaling and translation operations. When the mounting is elected as power of two, this sympathetic of wavelet transform is called dyadic orthonormal wavelet transform, which makes a way for discrete wavelet transform. The discrete wavelet transform (DWT) requires less space utilizing the space-saving coding based on the fact that wavelet families are orthogonal or biorthogonal bases, and thus do not produce redundant analysis. The DWT corresponds to its continuous version sampled usually on a dyadic grid, which means that the scales and translations are powers of two.

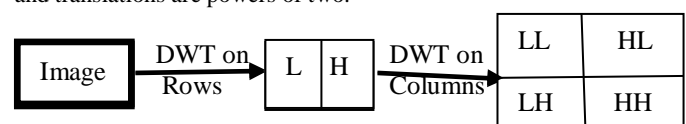


Fig. 1: Block Diagram of DWT

Digital Image Processing uses to denoising data from images and signals. There are many ways to denoise an image, which including gradient-based, sparse representation-based, non-local self-similarity-based, Gradient Histogram Preservative (GHP) algorithm. By using GHP algorithms noise can be removed, but it degrades the visual quality of an original image and also execution time to generate the denoised image is large. To avoid this problem, in this paper, we propose a Discrete Wavelet Transform (DWT), Stationary wavelet transform (SWT) and Singular Value Decomposition (SVD) method. This method is developed to enhance brightness and resolution while removing noise and execution time should be reduced. Our experimental results demonstrate that the proposed Discrete Wavelet Transform (DWT), Stationary wavelet transform (SWT) and Singular Value Decomposition (SVD) method can well preserve the texture

appearance in the denoised images and improve the resolution of denoised image and execution time should be reduced.

Sparsity Model: begin with an apparently naive discussion of sparsity in a ‘monoresolution’ model. Suppose that we observe an n-dimensional data vector y consisting of an unknown signal θ , which we wish to estimate, contaminated by additive Gaussian white noise of scale σ_n . If the model is represented in terms of its coefficients in a particular orthonormal basis B , we obtain (y^{Bk}) , (θ^{Bk}) , etc., though the dependence on B will usually be suppressed. Thus, in terms of basis coefficients,

$$y_k = \theta_k + \sigma_n z_k, \quad k = 1, \dots, n,$$

and $\{z_k\}$ are independently and identically distributed $N(0, 1)$ random variables. Here, we emphasize that $\theta = (\theta_k)$ is, in general, regarded as fixed and unknown. This model might be reasonable, for example, if we were viewing data as Fourier coefficients, and looking in a particular frequency band where the signal and noise spectrum are each about constant. It is assumed that $\{\theta_k\}$ are random, being drawn from a Gaussian distribution with $\text{Var}(\theta_k) = \tau_n^2$, then the optimal (Wiener) filter, or estimator, would involve linear shrinkage by a constant linear factor:

$$\hat{\theta}_k = \frac{\rho}{\rho + 1} y_k, \quad \rho = \frac{\tau_n^2}{\sigma_n^2}.$$

The ratio τ_n^2/σ_n^2 (or some function of it) is usually called the signal-to-noise ratio. The two key features of this traditional analysis are: (a) the Gaussian prior distribution leads to linear estimates as optimal; and (b) the linear shrinkage is invariant to orthogonal changes of coordinates: thus, the same Wiener filter is optimal, regardless of the basis chosen.

II. METHODOLOGY

The most general 1-D model for this is $s(n) = f(n) + \sigma e(n)$; where $n = 0, 1, 2, \dots, N-1$. The $e(n)$ are Gaussian random variables distributed as $N(0,1)$. The variance of the $\sigma e(n)$ is σ^2 , $s(n)$ is often a discrete-time signal with equal time steps corrupted by additive noise and you are attempting to recover that signal.

Generally, $s(n)$ as an N-dimensional random vector,

$$\begin{pmatrix} f(0) + \sigma e(0) \\ f(1) + \sigma e(1) \\ f(2) + \sigma e(2) \\ \vdots \\ f(N-1) + \sigma e(N-1) \end{pmatrix} = \begin{pmatrix} f(0) \\ f(1) \\ f(2) \\ \vdots \\ f(N-1) \end{pmatrix} + \begin{pmatrix} \sigma e(0) \\ \sigma e(1) \\ \sigma e(2) \\ \vdots \\ \sigma e(N-1) \end{pmatrix}$$

In this general situation, the relationship between denoising and regression is clear. This can replace the N-by-1 random vector by N-by-M random matrices to obtain the problem of recovering an image corrupted by additive noise.

For a broad class of signals and images that possess certain smoothness properties, wavelet techniques are optimal or near optimal for function recovery. Specifically, the method is efficient for families of functions f that have only a few nonzero wavelet coefficients. These functions have a sparse wavelet representation. For example, a smooth function nearly all over the place, with only a few abrupt changes, has such a property. The general wavelet-based method for denoising and nonparametric function estimation is to transform the data into the wavelet domain, threshold the wavelet coefficients, and invert the transform.

1) Soft or Hard Thresholding: Hard and soft thresholding are examples of shrinkage rules. The simplest scheme is hard thresholding. Let T denote the threshold and x your data. The hard and soft thresholding is

$$\eta(x) = \begin{cases} x & |x| \geq T \\ 0 & |x| < T \end{cases} \quad \text{and} \quad \eta(x) = \begin{cases} x-T & x > T \\ 0 & |x| \leq T \\ x+T & x < -T \end{cases}$$

2) Extension to Image Denoising: The denoising method described for the 1-D case applies also to images and applies well to geometrical images. A direct translation of the one dimensional model is $s(i, j) = f(i, j) + \sigma_e(i, j)$; where e is a white Gaussian noise with unit variance. The 2-D denoising procedure has the same three steps and uses 2-D wavelet tools instead of 1-D ones. For the threshold range, $\text{prod}(\text{size}(s))$ is used instead of $\text{length}(s)$ if the fixed form threshold is used.

3) 1-D Wavelet Variance Adaptive Thresholding: The idea is to define level by level time-dependent thresholds, and then increase the capability of the denoising strategies to handle non stationary variance noise models. Exactly, the model assumes that the observation is equal to the interesting signal superimposed on a noise. The Wavelet Denoising and Nonparametric Function Estimation is $s(n) = f(n) + \sigma e(n)$. Then again the noise variance can vary through time. There are several different variance values on several time intervals. The values as well as the intervals are unknown.

III. WAVELET FAMILIES

There are a number of basic functions that can be used as the mother wavelet for a wavelet transformation. Since the mother wavelet produces all wavelet functions used in the transformation through translation and scaling, it determines the characteristics of the resulting Wavelet Transform.

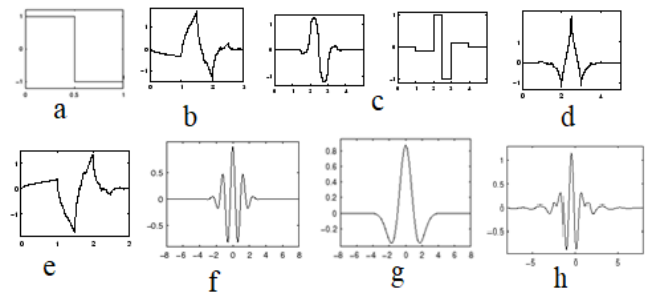


FIG.2: WAVELET FAMILIES: (A) HAAR (B) DAUBECHIES (C) BIORTHOGONAL (D) COIFLETS (E) SYMLETS (F) MORLET (G) MEXICAN HAT (H) MEYER.

IV. RESULTS

The one dimensional, thresholding histogram, values of thresholding, Original signal, soft and hard thresholding, recovery of signals and images are shown below figures.

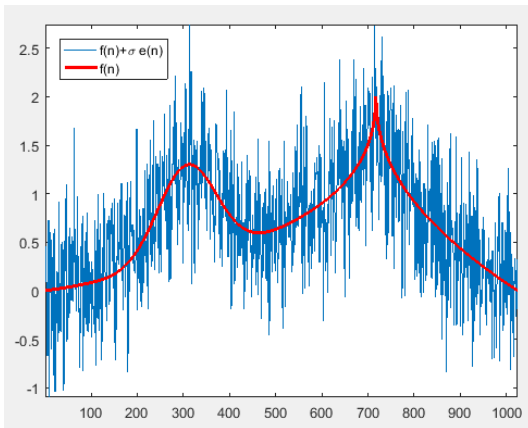


Fig 3: 1-D

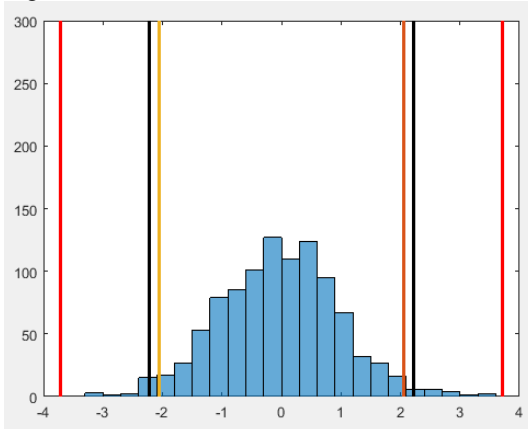


Fig 4: Threshold Selection

The values are:

thr_rigrsure =

2.0518

thr_univthresh =

3.7169

thr_heursure =

3.7169

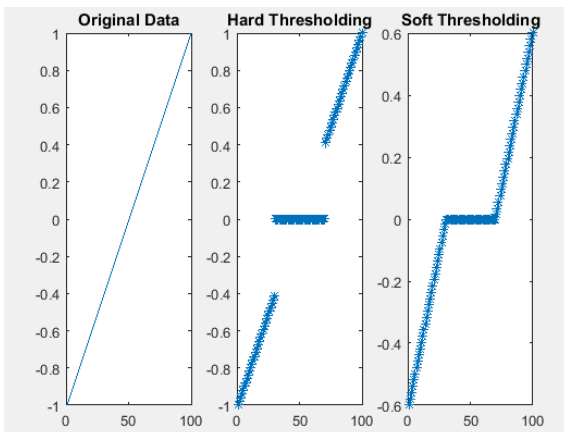


Fig 5: Original signal, soft and hard thresholding

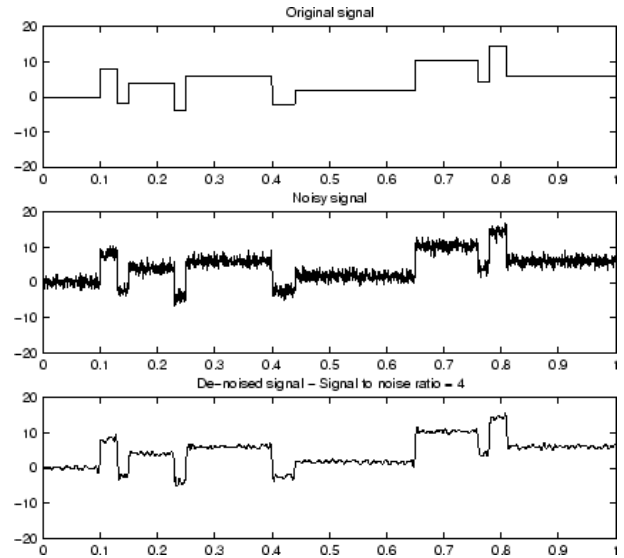


Fig 6: denoising signal

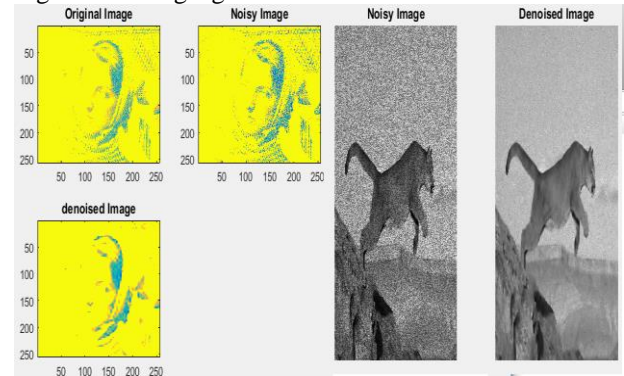


Fig 7: denoising Images

V. CONCLUSION

This paper provides real-world examples of signal and image enhancement and components detection using the wavelet transform. The data process are a real biomedical Electrocardiogram (ECG) signal and a spinal MR image. Detection of signal and image components can be utilised for their classification. The wavelet denoising methods offers high quality and give for the noise problem of signals and image. The acts of denoising methods for quite a lot of variations including thresholding rules and the type of wavelet were examined in the examples in order to put forward the suitable denoising results of the methods. The comparisons has finished for the three threshold estimation methods, wavelet types and the threshold types. The checks have showed that record important factor in wavelet denoising is what the decomposition level is rather than the wavelet type, threshold type or the estimation of threshold value.

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