



Peak to Average Power Ratio Evaluation of OFDM System with Randomness Limiting Process

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Abstract—Peak-to-average power-ratio reduction for OFDM systems is investigated in a probabilistic framework. A new constellation extension technique is developed whereby the data for each subcarrier are represented either by points in the original constellation or by extended points. In this paper, the analysis of Discrete Hartley Transform (DHT) precoded OFDM system using M-QAM (where $M=16, 32, 64, 256$) is performed. An optimal representation of the OFDM signal is achieved by using a de-randomization algorithm where the conditional probability involved is handled by using the Chernoff bound and the evaluation of the many hyperbolic cosine functions involved is replaced by a tight upper bound. The simulation results are based on DHT precoded OFDM system with DFT precoded OFDM system, Walsh Hadamard Transform (WHT) precoded OFDM system, Selected Mapping (SLM) based OFDM system and OFDM conventional. Simulation results show that the PAPR of DHT precoded OFDM system is lower than WHT precoded OFDM system, SLM-OFDM system and OFDM conventional and the work is even compared with De-randomization for number of rotation(R).

Index Terms—Discrete Hartley Transform, De-randomization, Orthogonal Frequency Division Multiplexing, Peak to Average Power Ratio

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier transmission scheme that has become the technology of choice for next generation wireless and wire-line digital communication systems because of its high speed data rates, high spectral efficiency, high quality service and robustness against narrow band interference and frequency selective fading [1]. OFDM thwarts Inter Symbol Interference (ISI) by inserting a Guard Interval (GI) using a Cyclic Prefix (CP) and moderates the frequency selectivity of the Multi Path (MP) channel with a simple equalizers. This leads to cheap hardware implementation. OFDM is widely adopted in various communication standards like Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), Digital Subscriber Lines (xDSL), Wireless Local Area Networks (WLAN), Wireless Metropolitan Area Networks (WMAN), Wireless Personal Area Networks (WPAN) and even in the beyond 3G Wide Area Networks (WAN) etc. Additionally, OFDM is a strong candidate for Wireless Asynchronous Transfer Mode (WATM).

A major drawback associated with OFDM is its large peak-to-average power-ratio (PAPR) which makes system performance very sensitive to distortion introduced by nonlinear devices such as power amplifiers (PAs) [2]. In an attempt to reduce the nonlinear distortion caused by the PAs, several techniques have been proposed that can reduce the PAPR of the OFDM signal before it enters a PA. A straightforward technique would be to limit the signal strength at the transmitter to a desired level through clipping but this technique degrades the bit-error-rate (BER) of the system and increases the out-of-band radiation [3][4] due to the increased harmonic content. Coding and band-pass filtering can reduce these effects but, unfortunately, they would increase the cost of the system [5]. A more efficient approach for the reduction of nonlinear distortion is through the use of PAPR-reduction algorithms and a variety of such algorithms have been described in the literature [6]. Very low PAPR can be achieved by these algorithms but at the cost of a significant reduction in the data transmission rate. Moreover, these algorithms require large look-up tables and, therefore, are more suitable for OFDM systems with a small number of subcarriers. A multiple signal representation approach has been proposed in [7][8] where a set of OFDM signals are generated at the OFDM transmitter and the transmit signal with the lowest peak power is selected. This approach is computationally efficient but it requires the transmission of a small amount of side information.

In [9], a symmetric constellation extension (SCE) algorithm has been proposed for PAPR reduction whereby the sub-symbols for each subcarrier are represented by two symmetric constellation points and an optimal representation has been derived by using a de-randomization algorithm. Since there is one bit that is not used to transmit any information for each constellation point, the transmit power of OFDM systems using constellation extension is much larger than that of OFDM systems with no constellation extension. In [10], PAPR-reduction algorithms have been proposed whereby the exterior points of the modulation constellation are modified using linear programming to reduce the peak power of the transmit signal. Due to the constellation extension, these algorithms require an

increase in the transmit power and computation complexity at the transmitter.

In this paper, a new algorithm for PAPR reduction is investigated in a probabilistic framework, which consists of two key ingredients. First, a new constellation extension technique is developed whereby the data for each subcarrier can be represented by a point in the original constellation or by an extended point. Second, a new de-randomization algorithm is proposed by applying the so-called method of conditional probability (MCP) and also the proposed algorithm is compared with the previous technique in the literature.

The paper is organized as follows. In Section II, a brief description of OFDM systems is given. In Section III, the proposed constellation extension scheme is described and new PAPR-reduction algorithms for the solution of the problem are described. In Section IV, describes about simulation results and finally conclusions are given in Section V.

II. SYSTEM MODEL

The OFDM system splits the high speed data stream into a number of parallel low data rate streams and these low rates data streams are transmitted simultaneously over a number of orthogonal subcarriers.

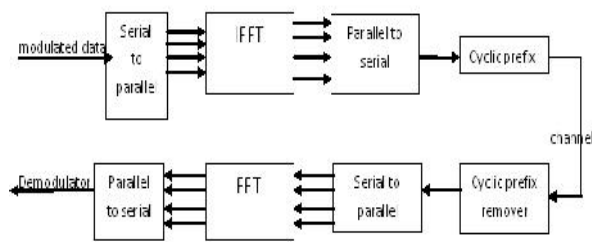


Figure1. Block diagram of OFDM system

Figure1 illustrates the block diagram of an OFDM system. Baseband modulated symbols are passed through serial to parallel converter which generates complex vector of size N. It can write the complex vector of size N as $X = [X_0, X_1, X_2, \dots, X_{N-1}]^T$, X is then passed through the IFFT block. The complex baseband OFDM signal with N subcarriers can be written as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \cdot e^{j2\pi \frac{n}{N}k} \quad (1)$$

Where $k=0,1,2,\dots,N-1$

Here $j = \sqrt{-1}$ and the PAPR of OFDM signal is calculated as

$$PAPR = \frac{\max |x_n|^2}{E[|x_n|^2]} \quad (2)$$

Where $E[.]$ denotes expectation and the Complementary

Cumulative Distribution Function (CCDF) for an OFDM signal can be written as

$$P(PAPR > PAPR_0) = 1 - (1 - e^{-PAPR_0})^N \quad (3)$$

Where $PAPR_0$ is the clipping level. This equation can be read as the probability that the PAPR of a symbol block exceeds some clip level $PAPR_0$.

III. PAPR REDUCTION VIA CONSTELLATION ALGORITHM

A. Constellation Extension Scheme

For the purpose of illustration, a 16-QAM modulation is assumed for each subcarrier in which case the constellation assumes the form shown in Fig. 2a. The constellation extension scheme for this type of modulation is illustrated in Fig. 2b where any data point with a value greater than or equal to 4 can be represented by a pair of two possible constellation points. For example, data point $D_k = 15$ (or 1111 in binary form) can be represented either by $X_k^0 = -3 - 3j$ or by $X_k^1 = -3 + 5j$. Where the superscripts of X_k^0 and X_k^1 are used to identify which constellation point is selected to represent D_k i.e., X_k^0 indicates that an exterior point of the conventional constellation is used to represent D_k on the other hand, X_k^1 indicates that a corresponding extended point is used to represent D_k . Based on this constellation extension technique, one seeks to reduce the PAPR of the transmit signals by selecting the optimal representation of data points by either the exterior or the extended points. Note that the minimum Euclidean distance between the extended constellation point and any conventional constellation point is guaranteed to be no less than the minimum distance among the conventional constellation points. As will be demonstrated later, the increase of average transmit power due to the constellation extension is fairly small and, consequently, the BER performance of the system will not be degraded significantly. Note also that the above constellation extension scheme can be easily applied to other modulation constellations without any difficulty.

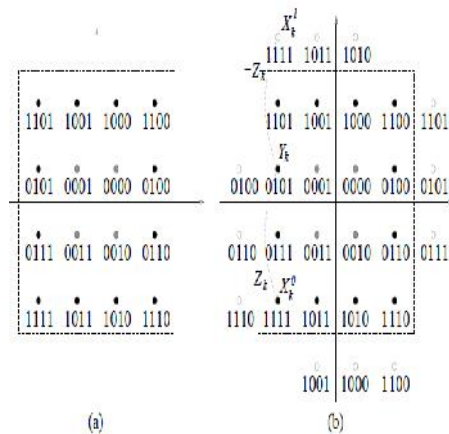


Figure. 2(a) 16-QAM Constellation with Gray code bit mapping. (b) Extension of 16-QAM constellation

B. PAPR-randomness limiting process:

The mini max optimization problem in [11] is an integer programming problem which can be solved by using the method of conditional probability [12][13] and a de-randomization algorithm based on the method of conditional probability will now be proposed. As will be shown, by applying the Chernoff bound to estimate the conditional probability involved and then replacing the many hyperbolic cosine functions produced by the Chernoff-bound estimate with a tight polynomial upper bound, a low computational complexity can be achieved.

I. Chernoff-Bound Based randomness limiting process:

The main results of the Chernoff-bound based de-randomization algorithm proposed in [14] are sketched. Consider sign vector $S=[S_1, \dots, S_K]$, where $s = s_1, \dots, s_K$ are treated as random variables that can assume the values of 1 or -1 with equal Probability. Let A_n^j be the event that $|c_n + \sum_{k=1}^k s_k d_{nk}| \geq j$ and $\Pr(A_n^j)$ be the probability that event A_n^j that occurs. Let us assume that j is chosen such that

$$\sum_{n=0}^{2N-1} \Pr(A_n^j) < 1 \quad (4)$$

If the first component of the optimal sign vector is taken to be $s_1^* = 1$, then a suboptimal sign vector s^* can be obtained sequentially as

$$s_j^* = \arg \left[\min_{s_j \in \{1, -1\}} \sum_{n=0}^{2N-1} \Pr(A_n^j | s_1^*, \dots, s_{j-1}^*, s_j) \right] \quad (5)$$

for $j= 2, \dots, K$. It can be shown that the sign vector $s^* = [s_1^*, \dots, s_K^*]$ obtained using (5) can be regarded as a suboptimal solution for which the objective function in the problem in [11] is guaranteed to be smaller than λ . Since numerical evaluation of the conditional probability is often difficult, an upper bound $U_n(j, s_1, \dots, s_j)$ was introduced in [14] for the conditional probability, known as *Pessimistic Estimator* [12], such that

$$\Pr(A_n^j | s_1, \dots, s_j) \leq U_n(j, s_1, \dots, s_j) \quad (6)$$

For $j = 1, \dots, K$ where the conditions

$$\min_{s_j \in \{1, -1\}} \sum_{n=0}^{2N-1} U_n(j, s_1, \dots, s_{j-1}, s_j)$$

$$\leq \sum_{n=0}^{2N-1} U_n(j, s_1, \dots, s_{j-1}) \quad (7)$$

and

$$\sum_{n=0}^{2N-1} U_n(j) < 1 \quad (8)$$

are satisfied. In effect, working with a pessimistic estimator $U_n(j, s_1, \dots, s_j)$ satisfying the conditions in (6)-(8), a suboptimal sign vector s^* can be determined sequentially as

$$s_j^* = -\text{sign} \left[\sum_{n=0}^{2N-1} U_n(j, s_1^*, \dots, s_{j-1}^*, 1) - \sum_{n=0}^{2N-1} U_n(j, s_1^*, \dots, s_{j-1}^*, -1) \right] \quad (9)$$

For $j = 2, 3, \dots, K$

By applying the Chernoff bound [15]

$$\Pr(Y \geq u) \leq e^{-xu} E(e^{xY}) \quad (10)$$

To the conditional probability

$$\Pr \left(\left| c_n + \sum_{k=1}^K s_k d_{nk} \right| \geq j \mid s_1, \dots, s_j \right) = \Pr \left(\sum_{k=j+1}^K s_k d_{nk} \geq j - c_n - \sum_{k=1}^j s_k d_{nk} \right) + \Pr \left(-\sum_{k=j+1}^K s_k d_{nk} \geq j + c_n + \sum_{k=1}^j s_k d_{nk} \right) \quad (11)$$

The modified equation is

$$\Pr \left(\left| c_n + \sum_{k=1}^K s_k d_{nk} \right| \geq j \mid s_1, \dots, s_j \right) \leq 2e^{-xj} \cosh \left(x c_n + \sum_{k=1}^j s_k d_{nk} \right) \prod_{k=j+1}^N \cosh(x d_{nk}) \quad (12)$$

Based on the above analysis, a pessimistic estimator can be derived as

$$U_n(j, s_1, \dots, s_j) = 2e^{-xj} \cosh \left(x^* c_n + x^* \sum_{k=1}^j s_k d_{nk} \right) \prod_{k=j+1}^N \cosh(x^* d_{nk}) \quad (13)$$

For $j=1, \dots, k$ where

$$v = \max_{0 \leq n \leq 2N-1} \left(c_n^2 + \sum_{k=1}^K d_{nk}^2 \right), \quad j^* = \sqrt{2v \log(4N)}$$

$$x^* = \}^* / V \quad (14)$$

IV SIMULATION

Figure.3 shows the comparisons of DHT-Precoder based OFDM system with DFT-Precoder based OFDM system, WHT-Precoder based OFDM system, SLM-OFDM (with V=2) system, OFDM-Original system and SR(R=1 and R=2) for N=64. For the proposed SR algorithm, the number of rotations is denoted as R. The rotation angle μ was set to the values $0, f/R, \dots, (R-2)f/R, (R-1)f/R$. At clip rate of 10^{-2} , the PAPR gain of 3dB, 2.5dB and 2dB is achieved when comparing DHT-Precoder based OFDM system with OFDM-Original system, WHT-Precoder based OFDM system and SLM-OFDM (with V=2) system respectively for 16-QAM. DHT-Precoding based OFDM system better than Selective Rotation technique.

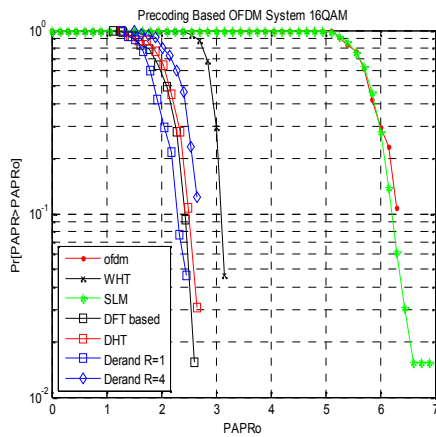


Figure3.Comparison of different techniques for 16-QAM

Figure.4 shows the comparisons of DHT-Precoder based OFDM system with DFT-Precoder based OFDM system, WHT-Precoder based OFDM system, SLM-OFDM (with V=2) system, OFDM-Original system and SR(R=1 and R=2) for N=64. For the proposed SR algorithm, the number of rotations is denoted as R. The rotation angle μ was set to the values $0, f/R, \dots, (R-2)f/R, (R-1)f/R$. At clip rate of 10^{-2} , the PAPR gain of 3.5dB, 2.7dB and 2.3dB is achieved when comparing DHT-Precoder based OFDM system with OFDM-Original system, WHT-Precoder based OFDM system and SLM-OFDM (with V=2) system respectively for 32-QAM. DHT-Precoding based OFDM system better than Selective Rotation technique..

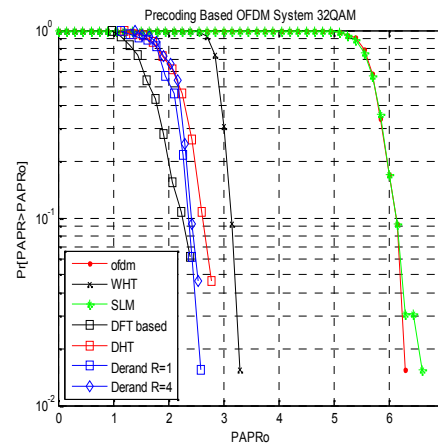


Figure4.Comparison of different techniques for 32-QAM

Figure.5 shows the comparisons of DHT-Precoder based OFDM system with DFT-Precoder based OFDM system, WHT-Precoder based OFDM system, SLM-OFDM (with V=2) system, OFDM-Original system and SR(R=1 and R=2) for N=64. For the proposed SR algorithm, the number of rotations is denoted as R. The rotation angle μ was set to the values $0, f/R, \dots, (R-2)f/R, (R-1)f/R$. At clip rate of 10^{-2} , the PAPR gain of 3dB, 3.2dB and 1.1dB is achieved when comparing DHT-Precoder based OFDM system with OFDM-Original system, WHT-Precoder based OFDM system and SLM-OFDM (with V=2) system respectively for 64-QAM. DHT-Precoding based OFDM system better than Selective Rotation technique.

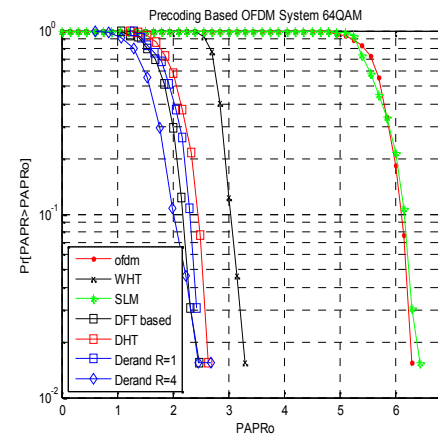


Figure5.Comparison of different techniques for 64-QAM

Figure.6 shows the comparisons of DHT-Precoder based OFDM system with DFT-Precoder based OFDM system, WHT-Precoder Based OFDM system, SLM-OFDM (with V=2) system, OFDM-Original system and SR(R=1 and R=2) for N=64. For the proposed SR algorithm, the number of rotations is denoted as R. The rotation angle μ was set to the values $0, f/R, \dots, (R-2)f/R, (R-1)f/R$. At clip rate of 10^{-2} , the PAPR gain of 2.4dB, 1.7dB and 1.8dB is achieved when comparing DHT-Precoder Based OFDM

system with OFDM-Original system, WHT-Precoder based OFDM system and SLM-OFDM (with $V=2$) system respectively for 256-QAM. DHT-Precoding based OFDM system better than Selective Rotation technique.

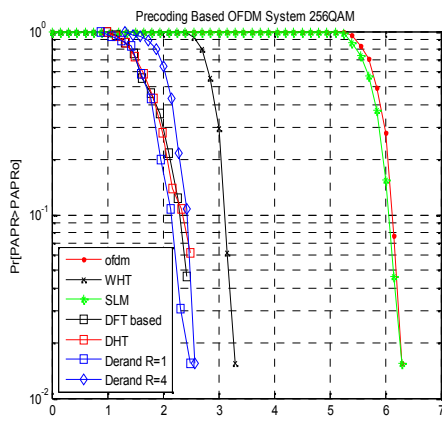


Figure6.Com

parison of different techniques for 256-QAM

V. CONCLUSION

A new PAPR-randomness limiting process for OFDM systems via constellation extension has been proposed based on a conditional probability method whereby the evaluation of the hyperbolic cosine functions involved in the Chernoff bound of the conditional probability is replaced by a tight polynomial bound that leads to reduced computational complexity. The peak to average power ratio is lower in precoding matrix as compared WHT-Precoder based SLM-OFDM, OFDM Original system and the constellation technique.

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